

Fraction Sangaku

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

?

$$\left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right) + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$$

$$= 1 + 1 + 1$$

$$= 3$$

$$\frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{5}{5} + \frac{6}{6}$$

?

$$\frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{5}{5} + \frac{6}{6}$$

$$= 1 + 1 + 1 + 1 + 1 + 1$$

$$= 6$$

$$\frac{1}{1} + \frac{-2}{2} + \frac{3}{3} + \frac{-4}{4} + \frac{5}{5} + \frac{-6}{6}$$

?

$$\frac{1}{1} + \frac{-2}{2} + \frac{3}{3} + \frac{-4}{4} + \frac{5}{5} + \frac{-6}{6}$$

$$= 1 - 1 + 1 - 1 + 1 - 1$$

$$= 0$$

$$\frac{1}{1} + \frac{-2}{-2} + \frac{3}{3} + \frac{-4}{-4} + \frac{5}{5} + \frac{-6}{-6}$$

?

$$\frac{1}{1} + \frac{-2}{-2} + \frac{3}{3} + \frac{-4}{-4} + \frac{5}{5} + \frac{-6}{-6}$$

$$= 1 + 1 + 1 + 1 + 1 + 1$$

$$= 6$$

$$\frac{1}{-1} + \frac{-2}{2} + \frac{3}{-3} + \frac{-4}{4} + \frac{5}{-5} + \frac{-6}{6}$$

?

$$\frac{1}{-1} + \frac{-2}{2} + \frac{3}{-3} + \frac{-4}{4} + \frac{5}{-5} + \frac{-6}{6}$$

$$= -1 - 1 - 1 - 1 - 1 - 1$$

$$= -6$$

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{6} + \frac{4}{8} + \frac{5}{10} + \frac{6}{12}$$

?

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{6} + \frac{4}{8} + \frac{5}{10} + \frac{6}{12}$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= 3$$

$$\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1 - 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5}$$

?

$$\frac{1}{5} + \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1 - 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - \frac{1}{5}$$

$$= \left(\frac{1}{5} - \frac{1}{5} \right) + \left(\frac{1}{4} - \frac{1}{4} \right) + \left(\frac{1}{3} - \frac{1}{3} \right) + \left(\frac{1}{2} - \frac{1}{2} \right) + (1 - 1)$$

$$= 0$$

$$1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$$

?

$$1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$$

$$= \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$$

$$= \frac{1}{4} - \frac{1}{8} - \frac{1}{16}$$

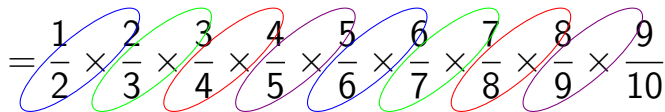
$$= \frac{1}{8} - \frac{1}{16}$$

$$= \frac{1}{16}$$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10}$$

?

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10}$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10}$$


$$= \frac{1}{10}$$

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$$

?

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$$

$$= \frac{\boxed{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} \times \frac{\textcircled{\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}}{\boxed{\frac{2}{2} \times \frac{2}{2} \times \frac{2}{2}}}}{= 1}$$

$$\frac{3}{789} : \frac{6}{789}$$

?

$$\frac{3}{789} \div \frac{6}{789}$$

$$= \frac{3}{\cancel{789}} \times \frac{\cancel{789}}{6}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\frac{789}{3} : \frac{789}{6}$$

?

$$\frac{789}{3} : \frac{789}{6}$$

$$= \frac{\cancel{789}}{3} \times \frac{6}{\cancel{789}}$$

$$= \frac{6}{3}$$

$$= 2$$