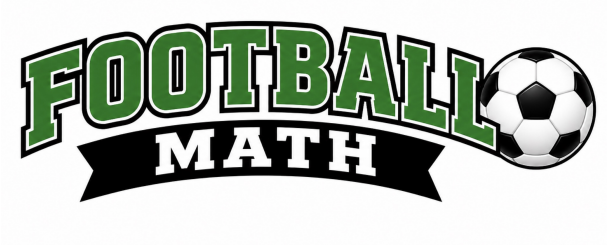

The International Football Association Board (IFAB) has official measures for the football pitch. There are two variants, *national* and *international*, but these only concern the touchline and the goal line. See the IFAB website `https://www.theifab.com/laws/latest/the-field-of-play/`.

(i) Consider the official IFAB measures of the football pitch in yards and the official IFAB measures of the football pitch in meters. Comment on the conversion that was made from yards to meters (recall that 1 yd=0.9144m).

From now on, we only consider the official measures in yards.

(ii) Consider the international variant. What are the areas of the largest football pitch, and of the smallest football pitch? Compute the percentage increase of the former compared to the latter.

(iii) The ratio between the lengths of the touchline and the goal line can vary. What is the largest (respectively, smallest) ratio that is allowed?

(iv) What is the maximal distance within a football pitch?

(v) How big is one goal area? How big is one penalty area? Now consider the football pitch in the international variant, with the intermediate width and length: Which percentage of the football pitch is within a goal area? Which percentage is within a penalty area?

(vi) Describe in full the penalty arc.

FOOTBALL MATH: SOLUTIONS

1. Some of the numbers have been obtained with the conversion factor, rounding up to the next centimeter. However, there are lengths (e.g. 18 yards) where the rounding was made to the next multiple of 5 centimetres. For the lengths of the goal line and the touchline, the rounding was made with errors that also exceeded 1 meter (however, no more than 2 meters), sometimes with a round-up and sometimes with a round-down: this was done so that the final measures are multiples of five meters. Finally, the corner arc is for simplicity 1 yard and 1 meter respectively.

2. The largest football pitch has an area of 9600 square yards. The smallest football pitch has an area of 7700 square yards. The percentage increase is then roughly 25%.

3. For the international variant, the ratio goes from $11/8 = 1.375$ to $12/7 = 1.714\ldots$. For the national variant, the ratio goes from 1 to 2.6. Ratio 1 means that in the national variant it is allowed to have a square football pitch.

4. The largest distance is the distance between two opposite corner flags, so it is the diagonal of the football pitch. Consider the biggest football pitch: in the international variant, this is roughly 144 yards; in the national variant, this is roughly 164 yards.

5. The goal area is a rectangle with area 120 square yards. The penalty area is a rectangle with area 792 square yards. For the given football pitch, the percentage of the areas given by the goal areas and the penalty areas are roughly 3% and 18% respectively.

6. The penalty arc is a circular arc of a circle whose radius is 10 yards. The distance from the chord to the center is 6 yards hence by Pythagoras's theorem the chord is 16 yards. In particular, consider the triangle consisting of half of the chord, one radius and the segment from the middle of the chord to the centre: this is a right triangle that is similar to the one with lengths 3,4,5. With trigonometry, one may compute that the central angle is twice the arccosine of 3/5, so it is roughly 106 degree. The length of the arc is then roughly 18.5 yards.

IFAB measures for a football pitch (yards)

The representation is not to scale

IFAB measures for a football pitch (meters)

The representation is not to scale