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Twelve Fantastic Stories

Riddles and Adventures for Young Mathematicians

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Before we begin…

What is mathematics? If we ask this question to children about to read these stories, they will probably talk about multiplication tables, exercises, calculations, and generally about formulas and rules they must learn at school. Because they think of their classroom experiences, they often see math as a bunch of rules and procedures.

Many people, adults and children alike, consider math to be the "hard" subject, the one that's hard to understand and might not seem useful—especially now that calculators and computers can do the math for us. But if you think about it, mathematics is like a "master key" that can open all doors. By teaching us to think and use logic, math helps us solve problems that aren't always about numbers. It helps us make good choices in everyday life, often without us realizing it.

But there's more: math gives us challenges and encourages us to develop helpful qualities like learning from mistakes, not giving up when things get tough, and having the courage and patience to

keep trying. These qualities, which math helps strengthen, are useful in all parts of life.

Most readers probably didn't expect to find math problems and their solutions hidden in the middle of a collection of stories. And they certainly didn't think math could be told like an adventure. But that's exactly what it is: math can be exciting, especially when we're not forced to "succeed" right away and can tackle challenges together, helping each other and sharing ideas.

In these pages, our young readers will travel with their imagination and think alongside the characters in the stories. These characters are all very different—funny and strange—but they share one thing: they all must solve tricky problems that seem impossible at first. These problems test our thinking skills and often lead to surprising, almost magical results.

By following these characters through their adventures, readers will be swept into unexpected situations: sharing a chocolate cake fairly, finding the exit of a maze, straightening picture frames, or timing things with hourglasses. Page by page, they'll discover not only that every problem has a solution, but also that imagination and teamwork are the keys to finding it.

Each of these twelve stories sets a small challenge by introducing a puzzle. You'll see the characters try different things—sometimes making mistakes or hitting dead ends—before they figure out the right approach. And at the end of each story, there's a short explanation to show which math ideas are hidden behind the puzzle and how they work.

We hope that by reading this collection, young readers will understand that math is much more varied and welcoming than most people think. And they'll see that solving a puzzle isn't just very satisfying—it can also become an exciting adventure!

Twelve Fantastic Stories

A CAKE FOR TWO

How will the dwarves Freddy and Teddy share a cake fairly?

On a hot, sunny afternoon, a large group of children is playing in a meadow. The echo of their shouts and laughter resounds among the park's trees, while some adults—presumably their parents—chat and exchange relaxed, amused glances. On the grass next to them, large picnic blankets are spread out, loaded with plates full of bread rolls, sandwiches, and various cakes, as well as glasses, napkins, and plates.

“Look at this splendour!” cries Freddy, a dwarf with rosy cheeks and a dark beard, observing the scene from a tree.

His companion Teddy nods in agreement, still lazily swinging back and forth on a branch. He seems tired and sleepy.

“Oh… look at those cookies… they must be chocolate ones, right? I would give anything to try one!” Freddy continues, his mouth watering, staring at a small platter on a checkered blanket.

“Maybe they will leave a few crumbs when they leave…” replies Teddy, not very convinced, still swinging.

“Crumbs?!? I don’t want crumbs! I want to eat those cookies!” exclaims Freddy angrily, standing up on his branch as if ready to spring into action. His stomach growls, also dissatisfied.

“We must not show ourselves to humans, that’s far too dangerous!” warns his friend, yawning. “We have to wait until they leave. We have no choice!”

Freddy grumblingly sits back down on the branch. But suddenly…

“Children, come here! The cake is here!” calls a voice from the trees.

A tall, brunette woman crosses the meadow and calls the children to her. In her hands, she holds a large, colourful cardboard box, which she carefully places on one of the picnic blankets. Kneeling down, she obviously intends to open it.

“Hurray! Yahoo!” the children shout enthusiastically, leaving their games and running loudly and somewhat disorderly towards the blanket.

“A cake? Did she really say cake?” Teddy abruptly stops swinging, and, forgetting his tiredness and laziness, stands on the branch to get a better view.

Then, pointing to the colourful box, he adds, “Look, Freddy, there must be a real delicacy in that box!”

But Freddy, upset and in a bad mood, does not respond. While the noisy group of children gathers around the brunette woman, a somewhat off-key singing can be heard: “Happy birthday to you! Happy birthday to you!” accompanied by a strong smell of chocolate.

“Do you smell that, Freddy? Do you smell that aroma?” asks Teddy excitedly, sniffing the air. “What wouldn’t I give to taste a bit of that cake…”

The gnome watches the scene dreamily. The chocolate smell becomes more and more intense and tempting.

“Oh yes, now I smell it too!” Freddy suddenly says, awakening from his lethargy, also sniffing. “It’s delicious, absolutely delicious!”

“Oh yes, nothing is tastier than a nice birthday cake. But the children won’t leave a crumb… you’ll see!” comments an old owl sitting on a nearby branch.

The two gnomes, too busy watching the party, hadn’t noticed her arrival. The poor owl is a bit annoyed, since the children’s games disturbed her midday nap.

The children gather loudly around the colourful box, which is now completely hidden by their heads.

“Please, children, don’t eat it all!” the two dwarves implore in chorus, feeling helpless.

“As if they’d leave a piece!” teases the owl at them. “They will eat everything, for sure!”

How could one stop this hungry horde of children? Deep inside, Freddy and Teddy know the owl is right: Not a single crumb of this chocolate treat will remain…

Disappointed, with growling stomachs and sad eyes, they remain silent, watching the overloaded checkered blanket. Suddenly, a loud “Braoum” tears the sky and makes the two gnomes start. All heads instinctively lift upwards, and worried looks and tense voices are heard all over the meadow.

“That’s thunder! There will be a thunderstorm!” say the people, running around.

In the rush and confusion, bags, platters, glasses, blankets, and gift packages are gathered up, while a second, even louder “Braoum” sounds in the sky. Thick raindrops start falling, more and more heavily.

“Now we also have a thunderstorm…” murmurs Teddy, looking depressed, as he sits back on his branch, staring at the dark clouds.

Under the trees, the frantic back-and-forth of the people continues for another thirty endless seconds. Until finally, on the now abandoned meadow, nothing remains except… a large, colourful cardboard box!

“Look, Teddy, that’s the cake! They forgot the cake!” exclaims Freddy happily, pointing to the precious box that the picnic-goers left behind in their hurry.

Hearing these words, Teddy's face lights up with joy and disbelief. The gnome stands up and looks towards the direction indicated by Freddy.

"Let's go get it!" they say together excitedly.

With nimble leaps from branch to branch, the two gnomes reach the ground and run as fast as they can towards their fragrant chocolate trophy. The box is large and seems quite heavy.

But despite its size, Freddy and Teddy will not be discouraged—especially given the prospect of a delicious chocolate cake.

"On three!" cries Freddy, grabbing one side of the box and giving Teddy a conspiratorial glance, who is already in position on the other side. "One, two, three…"

With combined efforts, the two gnomes lift the box and carry it back to their branch with quick, well-coordinated movements.

Now that they are safe and protected again, they can finally open the pretty, colourful box and admire what they have desired so much.

"Wonderful!" they cry, delighted.

In front of their eyes appears a whole cake. The black-handled knife with which the humans intended to cut it was also left in the box. Perfect!

“It’s ours!” shouts Teddy excitedly. “All ours!”

But after their initial enthusiasm, another feeling sets in. The two gnomes start giving each other suspicious looks.

“Alright, but… how do we share it?” Freddy asks warily.

“I saw it first when the woman brought it, so I get the bigger piece!” Teddy says seriously.

“Maybe you saw it first when she brought it, but I was the first to notice that the humans forgot it!” retorts Freddy, annoyed.

“Teddy saw it when they brought it, and Freddy noticed first that it was forgotten: The smartest decision would be to divide it evenly, don’t you think?” suggests the old owl, who continues watching them from her branch.

“Yes, of course…” mutters Teddy unconvincingly. Then, grabbing the knife and ready to cut, he adds, “I’ll do it!”

“Wait a second,” Freddy stops him, taking his arm and snatching the knife. “If you don’t mind, I’ll do it!”

“And why should you be the one to do it?” Teddy counters, annoyed.

“Because I’m sure you want the bigger piece!” Freddy replies, challenging him with his eyes.

“That’s not true!” Teddy protests indignantly. “You want the bigger piece!”

The two dwarves begin quarrelling over the box, each pulling on one side, while the storm grows stronger.

“Is this really the right time for a quarrel?” the old owl scolds them, shaking her head.

“He wants to cheat me!” Freddy accuses angrily.

“You want to steal from me!” Teddy retorts, offended.

“Can you be sure that no solution exists?” asks the owl calmly.

“No… unless we decide by lot who gets to cut the pieces,” murmurs Freddy.

“That’s an interesting idea. Are you sure that solution will bring peace?” the owl asks patiently.

“No, because I don’t trust him: if he cuts the pieces, he will take the bigger one for himself!” answers Teddy after some thought.

“You would do the same!” replies Freddy angrily.

The old owl sighs. “What if we find a solution that you both agree on?” she suggests.

“There is none,” says Freddy.

“There is none,” Teddy agrees.

And you? Do you think there is a solution that satisfies both Teddy and Freddy?

“Actually, maybe there is a solution…” the owl continues. “What if Freddy cuts the cake into two parts, and then Teddy gets to choose the piece he prefers? That way, Freddy, even if he cuts unevenly, knows that Teddy will choose first. So, he will be careful to cut the cake as evenly as possible. That should please you both. Does that sound like a reasonable solution?”

The two gnomes look at each other quietly for a moment, consider it, and finally realize the owl’s idea solves their problem.

“Is it alright if I cut?” asks Freddy finally in a friendly tone.

“And I choose,” answers Teddy, calmly, nodding. “Done!”

Under the satisfied gaze of the owl, the two gnomes shake hands. The long-awaited moment to taste the delicious chocolate cake has finally arrived…

What It’s About

*This story is about the problem of **fairly dividing a cake**. The two dwarves, Freddy and Teddy, do not want to fight anymore. The wise owl suggests: Freddy cuts the cake into two pieces, and Teddy chooses which piece he takes. This way, Teddy cannot complain. If both pieces are equal, he gets half exactly. If one piece is bigger, he will just take the bigger one—he has the choice. Freddy also cannot complain: he will try to cut the pieces as equally as possible. Because if he does, it does not matter which piece Teddy takes.*

The owl’s solution also works the other way around: Teddy cuts and Freddy chooses.

But what happens if there are three dwarves who want to share the cake? Then it gets more complicated. Still, there are strategies to solve that fairly!

Game theory *is a branch of mathematics that examines these kinds of strategies. It helps to find the best decisions in games. Some games are fair—all players have the same chances of winning. Others are not fair: for example, in chess, the player who moves first has a statistical advantage.*

Did you know that the first player in chess wins more often if they play the right moves?

If you want to try out a game to think about strategies, start with something simple like Tic-Tac-Toe. Challenge your friends or play on a computer!

If you get really good, try more difficult games like Othello. And if you want a real challenge, then try the Asian game Go.

But game theory is not only useful in games! It also helps in real life—everywhere you need to make decisions. For example: How much should you bid at an auction to get something at a good price? With which other team or party should you ally to become stronger together?

Game theory shows how to make smart decisions—whether in a game or in everyday life!

ESCAPING THE LABYRINTH

Emma is trapped in the labyrinth. Will she find the way out?

Blinded by a sudden light, Emma opens her blue eyes. Around her, grey stone walls rise up to the ceiling. Flickering torches cast a dim light along a long corridor.

Still half-asleep, Emma rubs her eyes and looks around, confused. She has no idea where she is and does not remember ever being in such a place. Or

does she? Wait… this stone corridor with torches reminds her of…

“Welcome!” A deep, rough voice behind her makes her jump.

Emma turns around and sees a large wooden door, flanked by two stone statues.

“Who spoke?” she asks fearfully.

“We did,” answers one of the statues, whose mouth moves as if it really speaks.

Although the statue is made of stone, it has a surprisingly human-like face. Emma is stunned. Where did she end up, and why are statues talking to her?

“We are the guardians of the Labyrinth of Mythos,” explains the second statue.

“A labyrinth?” Emma’s eyes widen. Of course! Now she remembers. This corridor, the torches, the statues… this looks exactly like the maze from her book about Greek myths.

Just a moment ago, she was lying in her room, holding a book about Daedalus and Icarus. And now? Now she stands in a dark, cold labyrinth, talking to stone creatures.

“You’re talking statues?” Emma asks hesitantly.

“We are not statues, but stone beings,” they correct her, somewhat offended.

Emma nods, though she doesn’t really see the difference. Instinctively, she approaches the wooden door to open it.

“Where are you going?” asks one statue.

“I want to go outside, home,” answers Emma.

“That’s not possible,” explains the other statue. “You can only return if you find the exit of the labyrinth. This here is the entrance. Of course, you can’t exit through the entrance…”

“And where is the exit?” Emma asks, growing more uneasy.

“On the other side,” say the guardians. “You must cross the labyrinth. That is your mission.”

“My mission?” repeats Emma, intimidated. “But… is there a Minotaur?”

“No, there are no monsters here. Only you,” one of the statues replies.

“Be careful! The labyrinth has no islands,” adds the other. “That means you can only return to the starting point if you turn back.”

“Now it’s time! Good luck!” exclaim the stone guardians, suddenly disappearing into the massive walls.

Emma stands there alone, unsure of what to do. She looks around as the torches cast shadows on the high walls. Finally, she dares to take a few steps, until she reaches a crossroads.

“And now? Which way should I go?” she murmurs desperately.

Suddenly a clear, bright voice rings out: “Why are you crying, little girl?”

Emma startles, looks around frantically, but sees no one.

“I didn’t imagine it!” she whispers, when suddenly something green jumps onto her arm. Instinctively, she screams and shakes her arm.

“Hey! What’s wrong with you?” protests the voice.

Emma looks more closely: in front of her sits a small green grasshopper wearing a tiny laurel crown on its head.

“Is it you who’s speaking?” Emma asks, unbelieving.

“Do you see anyone else around here?” the grasshopper replied.

“Forgive me for earlier,” Emma quickly said. “I didn’t mean to hurt you, really!”

“Apology accepted,” the grasshopper said with a smile, hopping back onto her arm.

There is no doubt, thought Emma, it *is* a real grasshopper—a green one. And it talks! First, the stone statues, and now a talking grasshopper... This place is definitely strange!

“Oh, how wonderful it is to have company! I was getting so bored here!” the insect exclaimed. Despite the circumstances, it seemed to be in good spirits. “What’s your name, little girl?”

“Me? I’m Emma. And you?”

“My name is Daedalus.”

“Daedalus?! Like the builder of the Minotaur’s labyrinth?” Emma rubbed her eyes in disbelief, recalling her mythology book and bedtime stories.

“Yes, that’s me, the very one who built the labyrinth!” the grasshopper declared solemnly.

“But… you’re a grasshopper,” Emma replied sceptically.

“Yes, I am now, that’s true,” Daedalus admitted. “But I was once a great architect and inventor.”

“Really? But what happened to you?” Emma asked, intrigued.

“Well... I got myself into a big mess,” the grasshopper said gloomily. “I angered the goddess, Athena. And now... here I am, trapped in this labyrinth. Forced to jump around in it forever.”

“That’s awful!” Emma said, her concern genuine. “I’m so sorry!”

After a moment of silence, she asked hesitantly, “Does that mean there’s no way out of here?”

“Of course, there’s a way out,” the grasshopper sighed. “But I can’t leave. Athena forbade me. Even if I did manage to get out, she’d just send me back. She’s done it before… more than once.”

“I see…” Emma nodded. “But what did you do that was so bad?”

“I was prideful,” he explained. “I claimed to be the smartest being that ever lived. And Athena didn’t like that at all. You see, the gods don’t take kindly to humans comparing themselves to them…”

“That’s terrible!” Emma exclaimed. “You were punished for something so small?”

Thinking for a moment, she added, “Wait... Are you saying the Olympian gods actually exist?”

“Shh!!! What are you doing, asking a question like that?!?” Daedalus hissed, clearly alarmed. “Do you want them to punish you too?”

Emma quickly shook her head as if trying to take back her words.

“Don’t worry,” the grasshopper said. “Anyway, it’s not so bad in here.” Then, with a hint of satisfaction, he added, “At least there are no cats or lizards!”

“Aren’t we lucky…” Emma murmured.

She was eager to leave this strange place. Pointing at the fork ahead of her, Emma asked, “Daedalus, could you tell me which way to go to get out of here?”

“Of course!” the grasshopper replied with an air of authority.

“Great! So which way do I go now?” Emma asked again, gesturing at the two paths. “Right or left?”
Daedalus smiled mischievously.

“I’ll let you in on a secret,” he whispered as if someone else might overhear. “There’s a strategy to

escape any labyrinth like this one, as long as it has no islands."

Emma's eyes widened in astonishment.

What do you think is the strategy to get out of the labyrinth?

The grasshopper clears his throat as if to prepare to speak.

"You must always keep your hand on the wall to your left. Sooner or later, you'll find the exit."

"Are you sure?" the little girl asks sceptically.

"One hundred percent guaranteed!" Daedalus replies confidently. "Keep your hand on the left wall and never let go. Are you ready?"

The girl nods, full of hope, and, brushing her fingers against the left wall, begins to walk with the grasshopper perched on her shoulder. After walking for a long time, Emma and Daedalus come to a break in the wall. The girl stops, uncertain.

"Now you need to take the turn on your left and continue following the left wall," the grasshopper instructs her. "Go on!"

The little girl eagerly resumes her walk. Then, after a few moments, she stops abruptly: the corridor they have taken is a dead end!

“What now?” Emma asks anxiously.

But Daedalus, still perched on her shoulder, remains unflustered.

“At the end of this stretch, the left wall turns into a small wall that blocks the corridor,” he explains calmly. “You need to keep walking and follow that wall. Then, as you can see, the small wall connects to the other wall of the corridor. You just have to keep following it on your left. You'll retrace your steps, but on the opposite side of where you came from.”

Emma nods and, following the grasshopper’s instructions to the letter, escapes the dead end.

“Good, now turn left,” Daedalus continues. “This way, you'll be taking a completely new path. That's the beauty of our strategy!” he adds, with a note of enthusiasm in his voice.

“It allows us to safely explore areas of the labyrinth we haven't yet been to, and little by little, it leads us through the entire maze. Eventually, we'll find the exit... Just wait and see!”

Following Daedalus' guidance, the girl moves from one branching path to another, from corridor to corridor, never taking her hand off the left wall.

After a long time exploring the labyrinth, Emma and Daedalus finally see the light shining from the exit. The little girl has no idea how long she has been walking. She only knows that she is tired and eager to go home to her parents.

"We made it! Hurray!" Emma exclaims, starting to run breathlessly toward the exit.

"Hey! Slow down! You're going to make me fall!" protests the grasshopper.

But his voice suddenly grows distant, like a faint echo.

"Daedalus? Are you there? Where did you go?" the girl asks, touching her shoulder with her hand.

But instead of the grasshopper, she hears the rustling of a sheet.

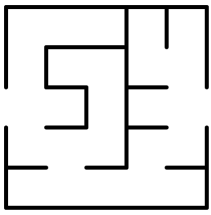
Opening her eyes, Emma realizes she is in the darkness of her bedroom. The book on Greek myths she had been reading just before falling asleep is still open to the picture of the labyrinth, with its two statues, high walls, and torches.

“I can't believe it! It was a dream!” she whispers before pulling the covers over her head and falling back asleep.

What It’s About

*In this story, we discuss the **problem of the labyrinth**. To make it simple, we imagine a classical maze. We don’t know exactly how the maze Emma is trapped in looks, but we know something important: there are no “islands.” That means you can only return to a place if you retrace your steps. Therefore, Daedalus’s idea works: just follow the left wall.*

If you want to try it yourself, you can put your left hand on the wall and follow it as you walk. Or you can draw a maze on paper and run your pencil along the left wall to see if the strategy really works



Note that following either the left wall or the right wall works equally well. You must choose one side. Also, if both the entrance and the exit of the maze lie on the outer boundary (which is often the case), the strategy will work even if there are "islands."

The areas of mathematics that help us develop such strategies for labyrinths are ***topology*** *and* ***graph theory****.*

Topology *is a more abstract version of geometry. It tries to distinguish shapes based on some of their base criteria.*

For example, consider a classic cube and do the following experiment: Count the number of faces, add the number of corners (vertices) where three faces meet, and then subtract the number of edges (the lines where two faces meet). The result is 2 (6 faces + 8 corners – 12 edges = 2). Try the same with a pyramid with a square base: the result is again 2 (5 faces + 5 corners – 8 edges = 2).

If you take any object—no matter how complex—and imagine that its surface is made up of triangles, squares, or other geometric shapes (like the black and white patches of a classic soccer ball), topology guarantees that the result is always 2, provided the object has no holes that go all the way through.

For objects with exactly one hole, like a donut or a mug with a handle, the result is always 0.

***Graph theory** studies "graphs," abstract objects made up of "nodes" connected by "edges." It's a bit like a railway network, where stations are "nodes" and the tracks are "edges," or like a road network, where intersections are "nodes" and roads are "edges."*

Although it sounds abstract, graph theory has many practical applications: it helps GPS systems find the best route and ensures data is transmitted efficiently over the internet.

Graphs can also represent other concepts, such as the phases of a process or possible moves in a chess game. In essence, every decision is like a fork in the road where you must choose a path.

As you see, mathematics and its concepts help us solve many practical problems—even escaping from labyrinths!

THE GOBLIN WHO LIES

Here is a tricky question for you: Imagine you meet two goblins. One always tells the truth, the other always lies. You do not know which one is the liar. Also, you can only ask one single question to get the right answer. What would you ask to ensure you get the correct information?

“We’re here!” cries Corry, placing her hand on the handle of a large door.

“Open it!” encourages Casper confidently.

The two friends have finally reached the last stage of their final exam. Behind this door is presumably the chest with their diplomas.

Corry depresses the latch, trembling with excitement. The room that opens up to them is small, dusty, and looks like some kind of forgotten storage closet. It's dimly lit, but despite the darkness and disorder, Casper and Corry recognize the large ebony chest they have been searching for.

"We made it! Hooray!" they call out together with joy.

To get this far, they had to solve riddles, tackle tricky tasks, and complete various skill tests—a long and tiring treasure hunt with clues leading them onward. But now they are almost at the goal. They just need to approach the chest and open it. But be careful: The Grand Master could have left traps…

"Darn! The chest is locked!" exclaims Casper, disappointed after trying in vain to open it.

"And where is the key?" asks Corry, at a loss. "All the clues led us here…"

"That means the key must be somewhere in this room!" concludes Casper, looking around.

The two youngsters begin thoroughly searching the room—rummaging through piles of books,

rolled-up carpets, and old tools. They whip up a cloud of dust.

“No one has cleaned here in ages…” complains Casper, coughing.

Corry nods with a dark expression. “I feel like I’m looking for a needle in a haystack…” she grumbles as well.

“What do you think, should we tell them?” a shrill, amused voice calls suddenly from above their heads.

Startled, Casper and Corry look up.

“Oh, they’ve noticed us!” says a second voice. “Look at those silly faces! They don’t look very clever. Do you really think they got this far without cheating?”

High above, near the ceiling, they see strange little beings: round faces, pointy ears, red pointed hats—and tiny wings that let them hover in the air.

“Who are you?” asks Corry suspiciously. “What are you doing up there?”

“We are your final challenge!” declare the gnomes mischievously, as one of them hovers down and looks Corry straight in the eye.

“The last challenge?!” stammers Casper. “I thought we had passed all the exams already…”

“You were mistaken,” teases the other gnome, who also descends from the ceiling.

“And what is this last challenge about?” the two friends ask desperately.

“Here it is!” exclaim the gnomes in unison, producing two almost identical keys—one gold and one silver.

One of the gnomes tosses Casper a rolled parchment sealed with wax. He carefully unrolls it while Corry watches closely.

“It’s a message from the Grand Master!” exclaims Casper, before reading aloud:

“*Dear students,*

If you have found this parchment, it means you have successfully completed all the tests of your final exam. Well done! Now only one last effort separates you from your diploma.

The gnomes before you, the brothers Mango and Tango, hold two keys: one golden and one silver. They will give you the key you name. Beware: Only one of the keys opens the chest containing your diplomas!

To find out which key is the correct one, you may ask only one question—and only to one of the two gnomes. The other gnome will not comment or correct the answer.

Be warned: I told one gnome to answer honestly to your question, while the other gnome will always lie. But you do not know which one is the truth-teller and which one is the liar.

Think carefully… and good luck!

The Grand Master."

Sighing, Casper and Corry exchange discouraged looks.

"Let's sum it up…" murmurs Corry after a moment. "We may ask only one question, and only to one gnome. If we knew which one tells the truth, we could just ask: 'Which key opens the chest?' and it would be easy. But we don't know who is honest. If we ask the liar this question, he will show us the wrong key."

"And if the liar… I don't know… Is there a way to unmask him?" wonders Casper, staring at the two floating figures.

"You mean…?!" responds Corry, eyeing the gnomes suspiciously.

The two tiny beings remain silent, but they look slightly tense now.

“They look exactly alike… They must be twin gnomes!” murmurs Corry finally.

“Yes, they are identical,” Casper agrees, sighing. “We’re back to square one.”

“Unfortunately…”

Discouraged, the friends sit down on the dusty floor.

“Are you giving up?” asks Mango with a mischievous grin.

Corry lifts her head and glares at him angrily.

“We’re not giving up!” she says firmly. “We just need time to think!”

“Ohhhh… they need time to think…” mocks Tango, fluttering around amused.

“Yes, we need time and quiet!” repeats Casper, glaring at him sternly.

“Be polite, Tango!” scolds Mango. “We must not help them, but we must not disturb them either!”

Corry and Casper nod and ignore the two brothers, whispering softly to each other.

“One of them will lie to our question, but we don’t know which one…” thinks Corry aloud. “Our

question must consider that. It cannot simply be: ‘Which key opens the chest?’ or the opposite.”

“Exactly. Such simple questions won’t do,” agrees Casper, thinking deeply.

“If we ask the honest gnome what his brother would say, he would honestly report his brother’s lie,” says Casper. “Thus, we’d get a false answer anyway…”

“Exactly!” exclaims Casper. “That means we can phrase our question so that both gnomes give the wrong answer! No matter who we ask, we would then know we must do the opposite.”

“You’re right!” Corry’s face brightens.

Casper smiles, feeling they are close to a solution. Then he hesitates.

“Wait a minute!” he exclaims, agitated. “You’re saying that if we ask the honest gnome what his brother would answer to a question, he’ll give us a false answer. But what if, instead of asking the honest gnome, we’re actually asking the liar? I mean… we have no way of knowing who’s who!”

“If we’re asking the liar…” Corry hesitates for a moment. “If we’re asking the liar, we know that, no matter what question we ask, he will always give us a false answer…”

“Yes, yes, that’s true! The lying gnome will never tell the truth,” Casper confirms. “If we ask him what his brother would answer to our question, he’ll always lie…”

“So, whether it’s the honest elf or the liar, they’ll both give the wrong answer!”

How do you need to phrase the question so that both gnomes give the wrong answer?

The gnomes continue to flutter around the ceiling, a bit bored, while Casper and Corry suddenly feel full of enthusiasm and energy, realizing that the solution is now within reach.
“So, the question we need to ask one of the gnomes is, *What would your brother say if we asked him which key is the right one?*” observes the young girl.

“And then, their answer will be the same and it will be... wrong!” continues the boy.

Corry nods. Then, to make sure she's found the solution to the problem, she tries to come up with a concrete example:

“Okay, so let's imagine the right key is the golden one. If we ask the lying gnome, *What would your brother say if we asked him which key is the right*

one? what would he answer? The lying gnome, the young girl continues, knows perfectly well that his brother would say the right key is the golden one. But, since he's a liar, he'll lie to us and say something like, *My brother would say the right key is the silver one.* On the other hand, if we ask the same question to the honest gnome, asking what his brother would say, he, knowing his brother is a liar, will honestly give the same answer: *My brother would say the right key is the silver one.*

“And the result is that both of them will point to the wrong key!” exclaims Casper.

“So, whatever the two gnomes answer, we have to understand the opposite,” concludes Corry.

“Yay! We did it!” the two friends cheer in unison, jumping up from the floor and starting to hop around the room, under the amused gaze of the two gnomes.

After much thinking and consideration, Casper and Corry can identify the right key and get their hands on the precious scrolls they've long desired. They have finally become Young Masters of Logical Sciences!

However, despite their success, they still haven't figured out which of the two gnomes is the liar.

Casper is ready to bet it's Mango, while Corry is convinced it's Tango... But, alas, they have no way of being sure—unless they start investigating again...

What It's About

In this story, we deal with a classic ***logic puzzle*** *about truth and lies. The puzzle relies on the idea: if you ask a truthful person what a liar would say, you get a lie; if you ask the liar what the truthful one would say, you also get a lie. Thus, both yield the same wrong answer to a carefully chosen question. By then inverting that answer, you find the truth.*

If you like such riddles, you can find many similar ones on the internet, often involving islands of knights (truth-tellers) and knaves (liars).

Logic *is the branch of mathematics that helps us understand how to form sentences that make statements either true or false. A statement cannot be both true and false at the same time.*

For example, we can use the word "no" to deny something: denying a true statement creates a lie, while denying a false statement leads to a correct one.

*If I say, "In this story, there are **no** gnomes," I'm lying. But if I say, "In this story, there are **no** fairies," I'm telling the truth. If I combine two true statements with "and," I'm telling the truth: for example, "There are gnomes **and** keys." If I combine two false statements with "and," I'm lying: for example, "There are fairies **and** elephants." However, if I say one true thing and one false thing, like "In this story, there are gnomes **and** fairies," I'm still lying.*

*When I use "or" to give a choice between two true things, my statement is true: for example, "There are gnomes **or** keys." If I give a choice between two false things, my statement is false: for example, "There are fairies **or** elephants." But if I give a choice between one true thing and one false thing, like "There are gnomes **or** fairies," my statement is true.*

*Sometimes, when I offer a choice, I can say that only one of the two options is possible: for example, "In this story, there are gnomes **or** keys, **but not both**." In this case, I'd be lying.*

Logic also teaches us how to figure out a statement based on another one (like with implications or logical deductions). For example, if I say I love all animals, then it's certain that I love

dogs too. After all, it's impossible to love all animals and not like dogs at the same time! There are so many more examples we could look at...

Remember: logic is important for all sciences and for any situation where we need to think clearly. It's also the foundation of how any computer, tablet, or smartphone works. It's everywhere!

THE CURIOUS MESSENGER

Imagine that you exchange letters with a friend, and you discover that the messenger who is supposed to deliver them reads them… What would you do to prevent it?

It is a day of celebration at Castle Bach. Prince Edmond and Princess Algebra are celebrating their engagement with their beloved parents—the rulers of Bach and the rulers of Boch—their friends and their families. Gentlemen and noble ladies have come from all corners of the two kingdoms to celebrate the young couple.

The kingdoms of Bach and Boch are neighbours and have always lived in peace and harmony. That is why the news of the engagement of Prince Edmond of Bach and Princess Algebra of Boch was greeted with much joy by everyone.

Between one dance and another, amid the whirl of festivities, the young engaged couple finally finds a moment of peace in the castle's lush garden.

"Oh, I am so tired!" exclaims the princess, smiling, her feet aching from all the dancing.

"So am I!" replies the prince in echo. "I am not used to all these pirouettes!"

The two young people laugh, sitting on a stone bench.

"I wanted to ask you something…" begins Edmond suddenly, hesitating.

"What is it?" Algebra looks at him with her big hazel eyes.

"Your last letters…" continues the prince. "Well, the wax seal with your crest seemed tampered with."

"As if someone had opened them and then tried to close them again?" the princess suggests, her face serious.

"Yes, exactly…"

“It happened to me too…” the princess admits with a sigh. “The last time I received one of your letters, a small piece of wax was even missing…”

“So someone has opened our letters?” Edmond concludes, more astonished than angry.

“Yes… more than once,” the princess agrees, nodding.

“But who could be interested in reading our messages?” Edmond wonders, thoughtful. “They are not state secrets…”

A smile appears on Algebra’s face.

“I think I know who…” she whispers.

Prince Edmond looks at her curiously, eager to hear her explanation.

“In the last letter I received,” explains the princess, fixing her large dark eyes in the green gaze of her companion, “not only was a piece of the seal missing, but there were also sticky honey prints on the paper…”

Prince Edmond’s eyes open wide with amazement.

“James?!?” he exclaims incredulously.

The princess nods.

“But why would he…” murmurs Edmond doubtfully, thinking of his trusted royal messenger, accused of tampering with and reading their letters?

“Because he is very curious… that’s why!” the princess replies. “Perhaps he is bored from going back and forth between Castle Bach and Castle Boch all the time. Reading our letters must be a distraction for him!”

“Yes, but boredom is not an acceptable excuse. What he does is wrong!”

“I agree with you,” admits the princess gravely. “That is why, despite his kindness and good heart, James deserves a little lesson…”

“What are you going to do?” asks Edmond, curious.

“This time, in addition to our usual small box, we will also use our padlocks!” replies Algebra, smiling cunningly.

The prince gives her a questioning look.

“Our padlocks?!?” he repeats, puzzled.

“Exactly: our personal padlocks. I have a plan!” announces Algebra firmly.

Leaning in close to the prince’s face, she begins to speak quickly into his ear. From time to time, Edmond responds quietly. They continue their consultation for a few minutes until the prince exclaims: “Perfect! We will do exactly that!”

Taking his fiancée’s hand in his, he heads smiling toward the castle’s ballroom.

Algebra and Edmond want to prevent the curious messenger from reading their letters. Each of them has a personal padlock and also the only key that can open it. What is their plan?

Two days after the engagement feast, early in the morning, Messenger James is summoned by Prince Edmond to the Blue Hall of Castle Bach.

"James, I have a mission for you," says the prince, smiling, while the chubby messenger enters the room and bows before him.

Presenting him with a small gold box, Edmond continues:

"I order you to deliver this box to Princess Algebra, at Castle Boch. Immediately!"

James observes the box, so small it could fit entirely in the palm of one hand. It is indeed the usual box, and the messenger is sure it contains a letter for the princess.

But for the first time, one of the two rings of the lock is closed by a padlock engraved with the crest of the royal house of Bach… Who knows why…

"Good! Now go!" the prince orders.

The messenger nods, bows, and quickly leaves the room. Edmond watches him walk away with a mischievous smile on his face.

Meanwhile, James sets out swiftly from Castle Bach. He knows the road well, since he has been traveling it several times a week for months now. Despite the unpleasant autumn drizzle, he reaches the high stone walls of Boch in less than two hours.

“A message from Prince Edmond for Princess Algebra!” he shouts to the guardian of the drawbridge.

The bridge is lowered, and James is admitted inside the castle walls. This time, however, unlike usual, he is not asked to hand over the box to a servant and wait in the courtyard. No, this time he is escorted directly to Princess Algebra, in the elegant Purple Hall of Castle Boch.

“A letter from Prince Edmond,” announces James solemnly, bowing and trying to hide his embarrassment.

He takes the small gold box out of his travel bag and hands it to the princess.

“Hello James, thank you!” greets Algebra with a smile, taking the small box in her hands.

The princess examines it attentively: on the gold padlock shines the eagle of the royal house of Bach. But instead of opening the box and giving James in turn a letter, she heads to her jewellery box. From one of its small drawers, she takes out a gold

padlock with the arms of the royal house of Boch and attaches it to the second ring of the box's lock. Then she hands the small box back to James.

"Good, now take it to Prince Edmond at Castle Bach," she orders in a kind but firm tone.

James looks at the box, not knowing what to think. Bring it back to Prince Edmond? Why so? The princess has not even opened it… Also, now it has two padlocks? What can that mean?

For a few moments, the messenger stands still and lost, eyes fixed on the box, mouth agape.

"James!" calls the princess kindly. "Is something wrong?"

The messenger startles. Then, stammering incomprehensible words, he bows quickly and leaves the room with the small box in his hands. Before putting it in his travel bag, he examines it carefully. On one side is the padlock from the prince, with the Bach crest, on the other side the padlock from the princess, with the Boch crest. Yet a single padlock would have been enough to close the box hermetically…

Also, who knows why, now he must bring everything back to Prince Edmond… With a shrug of resignation, James sets off again at a good pace toward Castle Bach.

For two hours, the messenger walks under the rain, wondering what is happening. By now it is lunchtime, and his stomach is growling loudly. This morning, in his haste, he forgot to pack his favourite snack in his travel bag: bread and honey…

He can't wait to arrive at Bach, deliver the box to Prince Edmond, and then head to the kitchen for a tasty meal prepared by the castle cook…

Absorbed in these appetizing thoughts, James arrives at Castle Bach. In a few minutes, he is admitted into the Blue Hall, in front of Prince Edmond.

"Back already?" welcomes the prince with a smile.

The messenger bows, nods, and hands him the box.

"Thank you!" exclaims the prince, seizing the box and looking at it with satisfaction.

"If you no longer need me, I will be going…" announces James, bowing.

"Wait!" Edmond stops him. "I still have another mission for you."

Taking a small golden key from his pocket, the prince opens the padlock with the Bach crest and removes it from the box. Then he hands the small box back to James.

The messenger watches the scene with eyes wide open. He can't believe it. His stomach is growling, reminding him it's lunchtime and he hasn't eaten in hours.

"What's wrong James?" asks Edmond innocently. "Is something bothering you?"

The messenger mechanically shakes his head, stammers something, bows, and flees the room as fast as he can.

A thousand thoughts race through his mind. This affair has become a real headache! Edmond first gave him a locked box to take to Algebra. Then Algebra, instead of opening it, added a second padlock and made him bring it back to Edmond. Now Edmond has removed his padlock and sent him again to Algebra… Why?

James is bewildered, and his stomach growls more than ever.

Again, box in bag, the messenger begins the two-hour journey toward Castle Boch. The unpleasant autumn drizzle continues to accompany him.

As he walks through forests and hills, ever more hungry and tired, James recalls all those times he peeked at the prince's and princess's letters to kill time. They were such romantic letters! They really

lifted his spirits! But now he is shuttling back and forth carrying a box locked once or twice…

Suddenly, an idea crosses his mind, making him see this strange day from a new angle. What if the prince and the princess realized he was reading their correspondence and are now punishing him, making him run back and forth in the rain and preventing him from having lunch?

After two endless hours of walking, exhausted, soaked, and starving, James finally arrives once again at Castle Boch. He is escorted again to Princess Algebra, in the beautiful Purple Hall.

“There you are again!” greets the princess with a smile.

The messenger bows, distressed and ashamed, and hands her the box. Princess Algebra takes it with satisfaction. Taking a key from her jewellery box, she removes the padlock bearing the Boch crest and opens the small golden box. Inside is a letter sealed with wax.

“Look what we had to do today to prevent you from reading our letters!” exclaims the princess, reproachfully. “I hope you understood the lesson and that you will never force us again to resort to such complicated methods!”

The messenger falls at her feet. He seems sincerely sorry.

“I beg your pardon, Princess. It will never happen again. I promise!” he says in a pleading tone, while his stomach growls loudly.

The princess nods, sympathetic.

“I believe you, James. Now, before leaving, go to the kitchen and eat something!” she invites him kindly.

Thus, while James is escorted to the castle’s kitchen, Algebra can finally sit down on her sofa and read the letter from her fiancé in perfect peace.

What It’s About

How to prevent someone from snooping in your correspondence?

The prince and the princess could have tried to make their letters unreadable by using a ***code language****, for example by using the letters of the alphabet differently from usual. Instead of writing “HELLO,” they could write “IFMMP,” replacing each letter by the next one in alphabetical order. But this trick is very basic, and James might have discovered it…*

Cryptography *is the branch of mathematics that aims to* ***encrypt*** *messages (i.e., make them readable*

only by the intended recipients) to secure communications.

This story figuratively describes a method of encryption. Using a personal padlock means encrypting the message so that only the person who has the key can read it.

It is also possible to place two padlocks independently, as in this story: it's like protecting a message with two passwords chosen and kept secret by two different people. Prince Edmond and Princess Algebra each have a personal padlock and each also has the only key to open it. This means that Algebra does not have the key to open Edmond's padlock, and Edmond does not have the key to open Algebra's padlock. That is why the plan they devised requires three trips for the message to be received. "That's a lot!" you might say, and you're right. But by doing so, the prince and princess ensure the sending is secure: their message is always protected by at least one padlock—or, in other words, by at least one password.

The situation would obviously be much simpler if there were only one padlock whose key both prince and princess possessed—like sharing a single password. But if you wanted secure communications between a large number of people, it would be

unthinkable to have a different padlock with shared keys for each person you want to communicate with.

A commonly used method for exchanging messages is called ***asymmetric cryptography or public key cryptography*** *(a name that can sometimes confuse us). Imagine we must encrypt a message for a friend, whom we will call Bob. If we apply the public key and private key method, it is Bob himself who publicly tells everyone how to encrypt messages destined for him. But at the same time, Bob is the only person who can* ***decrypt*** *(i.e., read and understand) messages encrypted in the way he proposed. In other words, it is as if Bob placed at everyone's disposal his mailbox (public), into which anyone can deposit mail. But only Bob has the (private) key to open his mailbox.*

Even if it is not obvious from this story, cryptography relies on mathematics.

Today, a message to be encrypted is first transcribed into a series of numbers. These numbers can then be manipulated using sophisticated mathematical techniques that guarantee, with a sufficient degree of security, that even a computer (or several computers working together) will not succeed in recovering the original message.

However, keep in mind that security is always relative. With enough time and/or computers to try all possible combinations, any password can be found.

Mathematics nevertheless assures us that if a password is long and complex enough, it would take all the computers in the world millions of years to find it by trial and error. In other words, such a password can be considered totally secure.

Cryptography is now widespread: almost all Internet communications use it. When you visit a website, send an email, write a message in an instant messenger, or make a purchase online, the data you transmit is very likely protected by cryptography.

TEA IN THE LIBRARY

The library rats Nico and Nina wonder: Is it possible to measure a six-minute interval using only two hourglasses, one of five minutes and the other of four?

As every Thursday, librarian rats Nico and Nina are cleaning and tidying their small but very rich library, which they have cared for many years. They love their job. They know all the books on the shelves and preserve them with great care and attention. Every Thursday morning, without

exception, they devote themselves diligently to cleaning all parts of the library.

They also know that they will receive this afternoon the visit of Mr. Mole, an elderly, learned, and erudite researcher whom they have known for a long time. He has written many books and is a longtime friend of the two librarians.

Besides being learned and erudite, Mr. Mole is also incredibly particular! Every Thursday afternoon, he knocks at the library door exactly at five o'clock, greets his librarian friends kindly, and stays with them exactly one hour. Not a minute more.

During this time, he chats amiably, tells anecdotes from his books, or mentions topics of his future research. And while he speaks, he savours a cup of his favourite tea: a rare infusion of nettle, lemon balm, calendula, and elderflower. The library rats are the only inhabitants of the land who have a well-stocked supply of it, as every year they receive it from a swallow friend returning from southern lands. Nico and Nina suspect that this rare and delicious tea is the main reason for Mr. Mole's weekly visits…

But let's get back to today. It's Thursday morning, Nico is carefully dusting a series of

hourglasses of various sizes displayed on a shelf in the library, wiping the glasses so they shine like crystal.

Meanwhile, Nina is cleaning one of the large carpets that cover the entire floor of the library, muffling noises and contributing to a cozy atmosphere. Suddenly, as the librarian hits the piece of fabric with a woven bamboo carpet beater, a cloud of dust rises in the room.

“Last week, we skipped cleaning the carpets, and here’s the result! More dust and more work…” comments Nina, coughing and slightly shaking her sweaty head. “We must never skip it again.”

Nico, not far from her, does not respond. The dust has climbed up his nostrils, causing an unpleasant tickle. Unable to hold it in, the librarian rat lets out an endless series of resounding sneezes.

“*Achoo, achoo, achoo*…” immediately followed by another endless series of deafening *sbong, bam, sbong, bam…*

Frightened, Nina looks in the direction of this racket: while Nico, perched on the shelf, continues sneezing, at least three of his precious hourglasses have crashed to the floor, scattering shards of glass and piles of sand everywhere.

“Oh, what a mess!” exclaims the female rat, immediately thinking of the trouble they will have to clean up again.

“Oh no! Our hourglasses!” moans Nico.

He has finally stopped sneezing and, observing the disaster from above, despairs, hands on his head and eyes swollen.

“Our poor, our so beautiful, our wonderful hourglasses…

“It’s truly a waste,” Nina replies, sadly echoing him.

“It’s worse than a waste, it’s a catastrophe! A real tragedy!” Nico keeps lamenting, climbing down from the furniture and looking around in dismay.

“Luckily, two hourglasses are safe…” Nina tries to console him. “Which ones survived?”

“Only the four-minute one and the five-minute one,” replies Nico, eyes reddened, looking at the two remaining hourglasses that, intact and shiny, still sit on the shelf. “We lost the ten-minute hourglass, the seven-minute one, and the six-minute one… Our poor hourglasses…”

“One moment…” murmurs Nina, worried. “You said… the six-minute one?”

“Yes, the ten, the seven, and the six…” repeats Nico inconsolably.

“Oh no! How are we going to do this evening?” asks the librarian anxiously.

“Do what?” The librarian, distraught by the loss of his hourglasses, cannot think of anything else.

“How will we prepare for Mr. Mole, explains Nina patiently. You know that his favourite tea must infuse for exactly six minutes…”

Nico suddenly turns pale. Nina is right: without the six-minute hourglass, it will be impossible to measure the infusion time. Mr. Mole will be very disappointed… He will never come back, that’s certain!

“I told you it’s a catastrophe, a tragedy!” he murmurs, devastated, while nervously starting to clean up the debris of glass and sand from the floor.

Nina helps him, working hard. The two librarians, worried and dismayed, gradually clear the floor of what remains of the hourglasses broken into a thousand pieces. When they are done, the library is as sparkling as ever. Their spirits, however, are at their lowest.

“What if we try to use the hourglasses we still have?” says Nina timidly.

Nico doesn’t seem convinced.

“We have a 4-minute hourglass and a 5-minute hourglass…,” he replies. “How do we measure exactly six minutes of infusion?”

Nina lowers her eyes, discouraged. She turns around to look at the two surviving hourglasses, so shiny, so beautiful.

“Nico, Nico, I have an idea!” she exclaims suddenly, as her face lights up. “I just got an idea to use both hourglasses…”

“I told you it’s hopeless…” grumbles Nico, leaning on a table, his head resting on his paw.

Do you think it’s possible to count six minutes with the two remaining hourglasses?

“No, on the contrary! Listen to this!” contradicts Nina, very excited. “If we start both hourglasses at the same time…”

“At the same time?” Nico, intrigued, lifts his gaze.

“Yes, at the same time,” Nina confirms convincingly. “When the 4-minute hourglass runs out, we will know there is exactly one minute left before the 5-minute hourglass empties. We must then start infusing the tea at that exact moment.”

“And then?” asks Nico, suddenly regaining hope.

“After one minute, the 5-minute hourglass will also have run out of sand,” continues Nina. “Then we will flip it immediately. When, after five minutes, it runs out again, a total of six minutes will have passed, and we can stop the tea infusion. What do you say? Might that work?”

A smile appears on Nico’s face. “I say you are a genius! A real genius!” he exclaims, spontaneously hugging her with his paws. “That way, we can measure exactly six minutes, just as Mr. Mole wants!”

“We won’t disappoint him, and we won’t look ridiculous…” rejoices the librarian, visibly relieved. “And he won’t stop coming to see us…”

“We are saved!” agrees Nico happily. Then, blushing, he adds: “Good thing that at least you remained calm!”

Nina smiles. Then she looks with a hint of melancholy at the shelf, which seems empty, stripped, and sad without the three broken hourglasses.

“I’m really sorry about the hourglasses,” she whispers.

“Yes, it’s really a pity,” Nico echoes. “I hope the traveling rat merchant will come back to see us soon so we can buy new ones. And when we have new

hourglasses, we will no longer have to resort to… alternative strategies to measure Mr. Mole's tea infusion time!"

What It's About

*In this story, there's a classic "**water pouring problem**" where you can make some measurements, but not all.*

Nina's idea is that with the hourglasses available, you can measure not just multiples of 4 and 5 minutes, or combinations of those, by using the hourglasses a few times. You can also measure exactly one minute, thanks to the difference between the two hourglasses. The countdown starts when the small hourglass runs out, and stops when the large hourglass empties too.

Nina's solution isn't just for measuring time, and it's not limited to using hourglasses. For example, imagine we have two bottles, one that holds 4 litres and another that holds 5 litres, and we need to measure 6 litres by pouring the liquid back and forth.

By using a similar method to the one in the story, we can measure exactly one litre by taking advantage of the difference between the two bottles. Pour the liquid from the large bottle into the small

one. When the small bottle is full, the liquid left in the large bottle will be exactly one litre. After emptying the small bottle and transferring the one litre from the large bottle into it, we can fill the large bottle with 5 litres again. This way, we'll have a total of 6 litres.

In general, puzzles involving hourglasses or pouring liquids often deal with ***natural numbers****, like 0, 1, 2, 3, and so on. Number theory (starting with simple arithmetic) is a huge part of mathematics, and it can answer questions about natural numbers.*

For example: what numbers can we get by subtracting a multiple of 5 from a multiple of 4? The answer is that you can get all numbers. For example, 22 is the same as six times 5 minus two times 4. And generally, you can find any number by calculating "n" times 5 minus "n" times 4.

However, you can't get all numbers by subtracting a multiple of 6 from a multiple of 4, because the sum or difference of two even numbers is always an even number.

If you like arithmetic, you can have fun learning about the ***greatest common divisor*** *of two (or more) numbers, or how to* ***break numbers down into prime factors****.*

Prime numbers *are special numbers (like 2, 3, 5, 7, 11, 13, 17, 19, 23…) that, when multiplied together, can make all other numbers. Here's an example you might know: 12 = 2 × 2 × 3. There are infinitely many prime numbers, and there are still questions about them that no one knows the answer to. For example, "Are there infinitely many twin prime numbers?" These are prime numbers that have a difference of 2 between them, like 3 and 5, 11 and 13, 17 and 19, and so on.*

One of the most fascinating things about number theory is that many of these ***open questions*** *seem simple, but no one has been able to answer them—yet.*

But you should never say never: in the coming decades, math will keep making great progress, and many of these unanswered questions will likely be solved!

THE SCALE OF FRIENDSHIP

Could you find a gold statuette among nine iron statuettes plated with gold, using only two weighings on an old-fashioned balance scale?

Or rather, she *used* to love it. That was until, at the start of the school year, a new girl arrived. Her name is Bianca, she has thick brown hair, and she's the same age as Ursule. And, just like Ursule, she really loves numbers. So much so that, sometimes, she even manages to get better grades…

This is a completely new situation for Ursule, who is not used to being outdone by anyone and hates being surpassed, which annoys her. Bianca seems to share the same attitude. The two of them just don't get along. They're constantly in competition, trying to prove at every turn that one is smarter and more capable than the other. It's a never-ending race to get the best results and the highest marks.

Their rivalry is known throughout the school, and even the class is divided into two groups: one side with Ursule's friends and the other with Bianca's. According to the teachers and the headmistress, it's time for the two girls to learn how to coexist without turning every situation into a competition. But how?

Unexpectedly, one October morning, the headmistress calls Ursule to her office. When she enters, all nervous, the girl is surprised and disappointed to find that Bianca is already there, sitting on a comfy little red velvet chair.

The two classmates barely greet each other, just a slight nod, each annoyed by the other's presence.

"Well," starts the headmistress, "now that you're both here, we can begin!"

With these words, she invites the girls to follow her into a room next to her office. Inside, one of the school assistants, Miss Dulys, is standing beside a table covered with a cloth.

With a swift gesture, the headmistress lifts the cloth, revealing an old two-pan balance and nine identical, shiny little statues. They're so shiny that they almost look like they're made of pure gold. The girls watch the scene with a mix of curiosity and suspicion, not understanding why they're in this room or what the headmistress expects from them.

"Dear Bianca and Ursule," says the headmistress. "You're probably wondering why I've called you here today."

Avoiding looking at each other, the two girls nod. "As you probably know, you are the best students in your class," says the headmistress with a warm smile. "That's why I'd like to give you a challenge… A challenge, mind you, that you will have to overcome together."

She pauses, looking up to observe the girls' reactions. They look clearly worried and exchange glances, their faces set with a look of stubborn defiance.
"As you can see," the headmistress continues, pointing at the table, "there is a two-pan balance

here, and nine little statues. They seem to be made of fine gold, don't they?"

Both girls nod.

"In fact, only one of them is truly made of gold," the headmistress goes on. "The others are made of iron coated with gold. Gold is heavier than iron, but the gold statue has a small cavity inside it. Therefore, the gold statue is only slightly heavier than the ones coated in gold. Is that clear so far?"

Both girls nod again, focusing on the headmistress's words.

"There's only one statue made of fine gold, the others are gold-plated iron," Ursula repeats carefully.

"And the one made of gold is a little heavier," Bianca quickly adds, not wanting to fall behind.

"Exactly!" comments the headmistress. "Your mission is to find the fine gold statue using this old two-pan balance. Do you know how it works?"

"Of course," Ursule answers promptly, giving Bianca a challenging look. "If we put two objects of different weights on each pan, the pan with the heavier object will go down, and the one with the lighter object will rise."

“While if we put two objects of the same weight, the two pans will stay balanced at the same level,” Bianca adds, returning Ursule’s gaze.

“Exactly!” the headmistress comments impassively. Then she adds, “To succeed in this challenge, you will need to follow specific rules.”

Impatient, the girls listen carefully.

“Rule number one,” the headmistress begins, counting on her fingers, “you cannot touch the statues. Only Miss Dulys is allowed to handle them.

Rule number two: You have the balance, that’s true, but you can only make two weighings. No more.

And finally, rule number three: You must work together. In this game, you win or lose as a team. So, I suggest you start thinking about the strategy you want to use to find the gold statue and win your reward.”

The girls stand still, carefully avoiding looking at each other. The headmistress watches them closely through her round glasses.

“Is there really a reward at stake, Miss Headmistress?” Bianca suddenly asks, intrigued.

“Everything in good time,” the headmistress replies sharply.

“And do we… have to participate?” Ursule asks hesitantly. She really doesn’t like the idea of working with Bianca and having to share a reward with her…

“You want to back out?” the headmistress asks, scrutinizing her face. “Why? Do you think you won’t succeed?”

“No, no!” the little girl quickly says, blushing.

“Any other questions?”

Ursule and Bianca shake their heads.

“Good,” concludes the headmistress with a satisfied air. “From now on, you have one hour. The clock on the wall will show you the time passed and how much you have left. Good luck!”

With these words, and after giving Miss Dulys a nod, the headmistress strides back into her office.

Which strategy would you use if you were in their shoes?

For a few minutes, Ursule and Bianca stand motionless in the middle of the room, unsure of what to do. Neither of them wants to make the first move. But time is passing inexorably.

“Okay, we can only do two weighings…” Bianca murmurs hesitantly. “So, if we compare two statues at a time, we’ll only find the gold one if it’s among the few we decide to weigh.”

Spreading her arms, she adds: “In two weighings, we can check only four of the nine statues. If the gold one isn’t among those four… well… we won’t have any way to figure out which one it is…”

“Well, it’s clear that this isn’t the right strategy…” Ursule replies coldly, without looking at her.

Bianca doesn’t respond but throws an offended glance at her classmate.

Long minutes of silence pass. The two girls really can’t accept the idea of having to collaborate… The inexorable ticking of the clock tells them that a quarter of an hour has already passed since the beginning of the challenge. Miss Dulys remains still and impassive next to the table, watching them silently.

Ursule, deep in thought, keeps her eyes on the ground: a part of her is trying to focus on the task to solve it, while another part wonders if it wouldn’t be wise to collaborate with Bianca, as the headmistress

asked. The problem is, they really can't stand each other!

What if… what if Bianca figured out the solution by herself? What if, at any moment, she raised her hand to say she has understood, that she knows what to do? That would be a terrible humiliation, that's what it would be!

Mechanically, Ursule shakes her head, making her braids sway. Bianca is in the same state, her gaze fixed, her facial expression hardened. In the room, only the ticking of the clock can be heard, reminding the girls that time is passing. The hour given for the task is gradually running out… Bianca swallows. Until now, her thoughts haven't led her very far. If only… if only she could discuss this with Ursule! But they are both far too proud and stubborn to admit they need each other.

Ursule sighs. She glances anxiously at the clock, then looks up at Miss Dulys's impassive face. Who knows what she thinks of them and this absurd situation…

"Time is passing…" murmurs Ursule hesitantly. "Do you have any ideas?"

“No,” Bianca sighs, shaking her curly head. But deep down, she’s relieved that Ursule has decided to make the first move.

“Let’s start with what we know,” suggests Ursule. “That means we can’t weigh them two by two since we only have two weighings. So, if we don’t get lucky in picking the statues to compare, we might not find the gold one. But there must be a strategy to identify it for sure.”

“Exactly,” Bianca agrees. “There must be a way to know for sure which one is the gold statue.” Then, after thinking for a few moments, she continues: “So… so, we’ll have to weigh them in groups?”

“Yes, that’s probably it…” Ursule reflects, staring intensely at the statues as if they could speak to her and give her hints.

Now that the ice is broken and they no longer have to worry about who will make the first move, Ursule and Bianca are calmer. With their minds freed from doubts, concerns, and fears, focusing and thinking is much easier…

“There are nine. It’s clear we can’t divide them into two equal groups…” Bianca rules out, her large black eyes fixed on the table.

Ursule nods.

“We could divide them into three groups…” Bianca suggests. “Three groups of three. What do you think?”

The clock keeps ticking. Looking up, the girls realize that thirty minutes have already passed: half of the time they have.

“If we weigh three statues on one side and three on the other…” Bianca murmurs.

“If the gold statue is in one of the two groups we compare, one will weigh more than the other, right?” Ursule continues. “So, the pans of the balance won’t be level.”

Ursule casts a hopeful look at Bianca, who nods in return. Their gazes finally meet.

“That way, we can know for sure which of the three groups contains the gold statue!” Bianca exclaims, excited. “If, when we compare two of the three groups, one is heavier than the other, then the statue must be in that one. Otherwise, if the pans stay balanced, that means the gold statue is in the third group, the one that hasn’t been weighed. Is that right?”

This time, it’s Ursule’s turn to agree, smiling broadly.

“So, once we know which group contains the gold statue, we can make the second weighing and compare the statues in that group!” Ursule adds, eager, feeling close to the solution.

“Exactly!” Bianca seems to share the same excitement. “We have three statues. If we compare two of them, two things can happen…”

“Either the pans stay balanced, and that means both statues are made of iron, or one of the statues is heavier. In the first case, the gold statue is the one that wasn’t weighed. In the second case, it’s the heavier one!” Ursule concludes enthusiastically.

“We’ve got the solution! We did it!” Bianca starts hopping, shouting with joy.

“Wait, we still have to find the statue!” warns Ursule. But she says it with a big smile on her face.

“Oops, yes, that’s right…” Bianca admits, theatrically putting her hand in front of her mouth.

And so, with Miss Dulys’ help, the two girls carry out their plan. First, they ask the teacher to place three randomly chosen statues on one pan of the balance and three on the other.

“None of these statues is made of gold,” they remark, noticing that the pans stay balanced. Then they add: “This means the fine gold statue is one of

the three statues we haven't weighed yet." They then ask Miss Dulys to place two of the remaining three statues on the two pans of the balance. The teacher complies, impassive and silent, placing one statue on each pan.

And suddenly, one of the pans starts to drop. "This statue is heavier!" the girls exclaim in unison. "That means it's the fine gold one! We found it!"

They point to the statue on the left pan, which is slightly lower than the other.

"I confirm, it's the heaviest one! Great teamwork!" Miss Dulys compliments them with a smile.

The two girls beam with satisfaction and high-five each other.

"Good thing you had the idea to separate the statues into three groups of three…" Ursule exclaims. "When you said that… when you said that, it really unlocked my brain!"

"Yes, but if you hadn't broken the ice first…" Bianca admits, blushing. "We'd still be silently staring at the ground, wasting time…"

"I can't believe we were so stupid!" Ursule comments, shaking her head.

The two of them laugh heartily. They are happy to have succeeded and are astonished to realize that by overcoming the challenge together, instead of doing it on their own, the moment hasn't been any less significant or less precious. On the contrary, having someone to share the joy and celebrate with is so much more fun!

It's as if the presence of another person multiplies the joy and excitement, intensifying it.

"Congratulations, Bianca and Ursule," says the headmistress, smiling, appearing in the doorway.

"I'm so happy that you've finally managed to put aside your rivalry and work together. I hope this will be a new beginning for you both. That was the reward: the chance to start over. Maybe you'll even become friends from now on, who knows?"

Well, who knows?

What It's About

At first glance, the problem may seem impossible to solve. But this story shows us that there is a solution.

Once we understand that we can compare more than two statues at a time, many possibilities open up.

To get useful information, it's important to put the same number of statues on each side of the scale. Since the weight difference between the real gold statue and the others is very small, if we put more statues on one side, the scale will tip, and we won't be able to learn anything new.

If during the first weighing, we put two statues on one side and two on the other, we leave five statues aside. If, by bad luck, the gold statue is among these five, we can't be sure which one it is since we only have one more weighing left. Trying to compare five statues with just one weighing creates many problems. For example, we could compare two statues against each other, but that wouldn't give us enough information about the remaining three. Or we could compare two pairs of statues, but that wouldn't help us tell which one in the heavier pair is made of real gold.

By changing the strategy from the start, we could choose to compare all nine statues by putting four statues on each side. If we did that, the gold statue could either be the only one left aside or it could be in one of the groups of four statues. But in the second case, we wouldn't be able to tell for sure which one it is with just one final weighing.

So, it's necessary to divide the statues into groups of three, like Bianca and Ursule did, so that after the first weighing, the gold statue will be in a group of just three statues, which will be easy to compare with the second weighing.

This story also shows that teamwork is powerful. When solving a tough problem, someone might have a good idea but struggle to turn it into a complete solution. It might then be someone else who picks up that idea and makes it work.

That's why so much scientific work is done in collaboration: sharing ideas, debating, and discussing them are really helpful for making progress. And let's face it, working together is much more fun!

If you love math, find someone to share your passion with. If possible, sign up for math competitions in your school or region (did you know there are even online competitions?).

This way, not only will you challenge yourself, but you might also meet people your age who share your love for math. And even though it feels great to win, remember that the most important thing is to participate!

Keep in mind that when preparing for a math competition, you may have to tackle new topics that you haven't studied in class yet. Everything you learn while practicing for these competitions, especially in the later years of high school, could be really useful if you ever decide to study at university.

If you're interested in math competitions, you can ask a teacher at your school for advice (and help). And... good luck!

THE TWO RABBITS

Mrs. Tortoise knows that both White Rabbit and Black Rabbit pass in front of her house several times a day. However, when she goes out into the garden, she almost always sees White Rabbit first. How is that possible?

Mrs. Tortoise is taking tea in her pretty flower garden, in front of her little house, right in the heart of the Red-Berry Woods. She likes to sit among her beloved plants, relax and enjoy the warmth of the sun's rays filtering through the trees.

Her days are always long—she wakes up well before dawn and goes to sleep well after dusk—and full of various activities: reading, cooking experiments, gymnastics, gardening, small repairs…

Despite her advanced age, Mrs. Tortoise is full of energy and has many interests!

This afternoon, Mrs. Tortoise is in the company of her dear friend, Mrs. Badger, who has just returned from a long journey. Having recently retired after working many years at the Institute of Meteorology of Red-Berry Woods, Mrs. Badger treated herself to a long rest by the sea, in some exotic place. She returned home refreshed, with a host of stories to tell her best friend!

While the two old ladies chat amiably, White Rabbit, the young courier of the Meteorology Institute, appears on the road that, crossing the woods, runs along the fence of the little house.

The rabbit greets Mrs. Tortoise and Mrs. Badger warmly and walks briskly along the path. He is carrying a rolled sheet of paper to deliver to the Institute located a few hundred meters from Mrs. Tortoise’s house.

“The afternoon weather bulletin must have arrived,” comments Mrs. Badger casually while continuing to nibble delicious cinnamon biscuits.

Mrs. Tortoise follows the rabbit’s silhouette with her gaze until he disappears among the trees.

“I see him pass every day,” she notes. “And also the other youngster, Black Rabbit…”

After a few moments, she adds, pointing to the road: “In fact, here he comes over there!”

Black Rabbit approaches, hopping along the path. He greets the two ladies cordially and continues on his way to the Meteorology Institute. He too holds a rolled sheet of paper.

“That must be the other late-afternoon weather bulletin,” says Mrs. Badger, after smiling at the young rabbit’s greeting. “Whoever wants to go to the Institute must pass by your house…”

“I don’t understand,” murmurs a puzzled Mrs. Tortoise. “How many weather bulletins are there per day?”

“The Meteorology Institute receives regular information from two observation stations, one located to the south of Red-Berry and one to the north,” explains Mrs. Badger. “Each day, it receives five bulletins from each of the two stations: two in

the morning and three in the afternoon. White Rabbit is in charge of delivering the bulletins from the north station, and Black Rabbit brings those from the south station."

"Really? I didn't know that!" exclaims Mrs. Tortoise, surprised. Then, curiously, she asks: "And where exactly are these stations? Are they far away?"

"No, no, they are not far. They are both half an hour's journey from here. If, when leaving your house's gate, you turn in the opposite direction from the Institute, the road leads you out of Red-Berry Woods to a small crossroads. From there, one path leads to the north station and another to the south station."

"I understand," murmurs Mrs. Tortoise.

Although she has always lived in Red-Berry, she has never ventured outside the woods. The two friends remain silent. Mrs. Tortoise's forehead is furrowed, as if very focused.

"You said that White Rabbit and Black Rabbit each must bring a bulletin to the Institute five times a day?" she asks after a moment.

"Yes," nods Mrs. Badger, still munching biscuits.

“That means that each of them passes in front of my house about ten times a day…” continues Mrs. Tortoise.

Mrs. Badger nods again.

“Yes, the two rabbits must bring their bulletins to their destination and then return each to their own station, waiting for the next bulletins.”

“I see…” says Mrs. Tortoise.

“Is something wrong?” asks Mrs. Badger, noticing her friend’s thoughtful expression.

Mrs. Tortoise sighs, pulling herself together.

“Well, I was thinking… Something intrigues me and I can’t explain it…” she begins hesitantly, searching for her words. “If both rabbits pass ten times a day in front of my house, both quite regularly… then why, when I go out to the garden at any random hour in the morning or afternoon, do I almost always see White Rabbit passing by first?”

“Maybe you and White Rabbit share the same schedule…” suggests Mrs. Badger vaguely.

“But I have no schedule!” giggles Mrs. Tortoise. “I come out for all sorts of reasons: repotting flowers, drinking tea, picking tomatoes, repainting the fence… But whenever I’m in the garden, I

almost always see White Rabbit before Black Rabbit. Don't you think that's strange?"

"Yes, indeed, it's a bit odd…" acknowledges Mrs. Badger. "Maybe it's just a coincidence?"

"But they pass ten times a day!" insists Mrs. Tortoise, not convinced by the mere idea of coincidence.

Mrs. Badger, thoughtful in turn, puts down her teacup.

"So," she explains, recalling her many years working at the Meteorology Institute. "The bulletins must be sent about every two hours, between 7 a.m. and 5 p.m. Usually, the bulletin from the north station leaves first. Then, shortly after, the one from the south station. This is because the north station takes its measurements a minute or two before the south station."

Mrs. Tortoise listens attentively and with interest.

"White Rabbit is responsible for transmitting the bulletin from the north station. He goes to the Institute and then returns to his station. Meanwhile, Black Rabbit, starting from the south station, also delivers his bulletin to the Institute and returns to his station."

“All this does not explain why I almost always see White Rabbit first…”

The two ladies remain silent for a few minutes.

“In fact, it does!” suddenly exclaims Mrs. Badger, as if she had a revelation. “It’s now obvious!”

What do you think is the solution to this mystery?

Mrs. Tortoise looks at her, still puzzled.

“What is obvious?”

“What’s obvious is why you always see White Rabbit first!” replies Mrs. Badger, smiling with satisfaction. “Both rabbits pass ten times a day in front of your house, but White Rabbit always passes a minute or two before Black Rabbit.”

Mrs. Tortoise nods silently.

“As for you, you come out into your garden several times a day at different times,” continues Mrs. Badger. “To see Black Rabbit first, you would have to step into the garden precisely during that one or two-minute interval that separates White Rabbit’s passage from Black Rabbit’s. Yet, if you think about it, it is much more likely that you come out at any

other time rather than precisely in that very short interval of one or two minutes. Don't you think?"

Mrs. Tortoise remains silent for a few moments, as if to let her friend's words sink in.

"Now that you mention it…" she murmurs, nodding her head. "It's true! It's much less likely that I come out into the garden at a precise short interval of time rather than at any other moment. Therefore, I'm more likely to see White Rabbit first!"

"Indeed!" smiles Mrs. Badger. "It seemed like there was no explanation, but actually, there is a simple one!"

Amused, the two friends laugh, satisfied with the conclusion they reached. They return to enjoying their snack. Meanwhile, White Rabbit reappears on the path after delivering the bulletin to the Institute, briskly heading back toward the north station.

What It's About

In this story, reasoning helped Mrs. Badger understand how a seemingly inexplicable fact actually has a simple explanation.

If we think carefully, this situation often happens when solving riddles. After discovering the solution,

we might say: "But of course! Why didn't I think of that before?" Don't worry if you fail to solve a riddle; you're not the only one! Many people can't solve the hardest riddles. Once you know the solution, try to understand it. If you have doubts, talk about it with someone who can help.

Maybe you are still too young to understand something, meaning you have not yet learned the necessary tools in school. In that case, you are at a disadvantage compared to older folks—but not less intelligent!

The puzzle in this story deals with probability. ***Probability theory*** *in mathematics studies how likely events are. Sometimes, it's surprising to see that one event is more likely than another.*

For example, here are two statements that seem contradictory but are both true: Most children have siblings, and most families have only one child.

To understand this, imagine there are 20 families: 12 have only one child, while 8 have two children. Thus, there are more families with a single child (12 vs. 8), but there are more children who have at least one sibling (16 vs. 12, since 8 families have 2 children = 16 children, and 12 families have 1 child = 12 children).

Another famous example is the "birthday paradox." In a group of people, how likely is it that two of them share the same birthday? With at least 23 people, the probability exceeds 50%, even though each person's birthday could be any of 365 days. (To simplify, we ignore leap years and assume each day is equally likely.)

TRAVELING WITH CAT, DOG AND MOUSE

If the cat cannot be left alone with the mouse, and the dog cannot be left alone with the cat, what will Witch Drusilla do? Deprived of her magic wand and only able to transport one animal at a time, how will she manage to organise her journey to the International Magic Fair?

A big cauldron simmers over the fire in the fireplace, under the watchful eye of a black cat, while a brown-furred dog sleeps peacefully on a cushion not far away. In a corner, a grey mouse happily nibbles on a piece of cheese.

Suddenly, there is a knock at the door.

The cat starts meowing. The dog, without waking, growls softly and turns his muzzle towards the wall. As for the mouse, it continues to eat, focused on its cheese.

“I’m coming!” cries a young, shrill voice.

It’s the mistress of the house, a witch with dark hair, pale hands, and long fingers. She stops her current activity—dissolving a yellow powder into a green liquid—and heads quickly to the door, knocking over some boxes left on the floor in the process.

“Good evening, Drusilla. Am I disturbing you?” A stout woman dressed in dark colours appears on the doorstep.

It’s Domitilla, the neighbour: a remarkable seer, able to read the future in crystal balls, in coffee grounds, and even at the bottom of dishes… though, to be honest, her predictions don’t always come true.

Domitilla carries a wicker basket.

“I brought you dinner!” she exclaims cheerfully, placing the basket on the table.

Then, looking around in amazement, she adds: “By all the devils! What is this mess? How can you work like this?”

Drusilla shrugs, returning to her test tubes and alembics. In recent weeks, she has had little time for tidying up and cleaning. If not for Domitilla and her carefully prepared meals, she might have even forgotten to eat…

The International Magic Fair starts in just three days, and the young witch wants to be sure all her recipes are perfect, ready to be presented and sold. The Fair is one of the most important events in the magical world. It takes place every four years, each time in a different city, and attracts mages and witches from all over the planet. Drusilla knows it's her big chance: she can finally show her creations, her philtres, and potions to the whole world.

"What's that strange smell… What's cooking in there?" asks Domitilla, approaching the fireplace and pointing to the big cauldron.

The cat, disturbed by the woman's movement, takes refuge in another corner of the room, meowing in protest.

"Rhubarb extract, lemon juice, ginger, termite eggs, viper venom…" Drusilla enumerates absentmindedly, without looking up from her work.

“But that’s a disgusting mixture! And what could something like that be good for?” asks Domitilla, pinching her nose.

“It’s a rejuvenation potion…” replies Drusilla.

“Rejuvenation? Really?” Domitilla suddenly seems interested. “And… does it work?”

“Yes, of course it works!” confirms Drusilla. “It could make anyone become a child again…”

“Become a child again?” Domitilla looks doubtful. “But who would want to become a child again?!?”

The seer sighs. She knows her young friend is very skilled at creating new and effective potions. But sometimes it seems that Drusilla wastes her time on projects that make no sense…

“Which potions will you present at the Fair?” asks Domitilla, examining the various containers scattered around the room.

“I want to bring all those I’ve tested and perfected. In total, there are 53. And they must all fit in there,” replies Drusilla, indicating a wooden chest on the floor.

It’s a medium-sized crate, rather ordinary in appearance, except for a wand shooting sparks

depicted on one side: the unmistakable logo of the International Magic Fair.

“You’ll be very loaded… Can you carry it all by yourself?” Domitilla inquires again.

Raising her head, Drusilla nods.

“Yes, the wooden crate can be transported on the broom.”

Then, sighing, she continues: “The problem, however, is Grey, Coaly, and Cocodog. You know I cannot leave them home alone. And I cannot entrust them to anyone else, because they would suffer too much from separation. They are not used to living without me…”

Domitilla, now serious, agrees, turning to look at Cocodog the sleeping dog, Coaly the black cat with big emerald green eyes, and Grey the gourmand mouse—Drusilla’s cherished pets.

Three little angels, bound to their mistress by affection and trust. But whenever she leaves, they pounce on each other.

Coaly the cat always tries to eat Grey the mouse. And Cocodog the dog always attacks Coaly the cat. Leaving them unsupervised is impossible: they would hurt each other. Nor would any of Drusilla’s friends agree to take care of them in her absence…

“You could always travel with a spell,” suggests Domitilla, surprised that her witch friend hadn’t thought of it.

Drusilla shakes her head, discouraged. “No spells, alas. Yesterday, my wand fell by mistake into one of the cauldrons and… well… boom… it exploded!” she says, making a gesture with her hands.

“Exploded?” repeats Domitilla, astonished.

It’s the first time, in so many years among witches and wizards, that she hears about an exploding magic wand…

The seer sighs, shaking her head slightly. In her life, she never knew a clumsier witch than Drusilla… She’s very intelligent, yes, but also incredibly awkward… How does she always get into these messes?

“I’ll buy a new wand at the Fair,” Drusilla continues, lost in thought. “Perhaps a newer model and a bit sturdier…”

“Yes, of course, a good idea,” Domitilla agrees, sighing.

Then, looking at the chest, the broom in a corner, and the three animals, she asks: “Besides the chest, can you carry the animals on the broom?”

“With the chest, I can only take one at a time,” murmurs Drusilla.

“Then you’ll have to make several trips. You could transport first the chest and one animal,” Domitilla suggests, “then leave them at the Fair and return for the other two animals.”

Drusilla shakes her head again. “Oh no, I can’t leave the chest unsupervised!” she objects vehemently. “Inside is all my work. I don’t want it lost or damaged. The chest must always stay with me, safe!”

“Then you’ll have to make three trips…” tries Domitilla again.

“Alas, it’s not that simple. I cannot leave Grey alone with Coaly, and I cannot leave Coaly alone with Cocodog,” Drusilla reminds her.

Domitilla becomes pensive: Grey the mouse cannot stay with Coaly the cat. But Cocodog the dog cannot stay with Coaly the cat either…

“By all the devils! There must be a solution!” exclaims the seer impatiently. “Let’s say you take the mouse Grey first… Coaly and Cocodog remain alone… Oh no… these two can’t be left alone either! Let’s try again: you take the cat Coaly first. Grey and Cocodog remain waiting at home and

behave. Then you come back and take Grey the mouse to the Fair and leave him there…"

"With Coaly?" adds Drusilla doubtfully.

"Oh no… you're right, that's not possible!" Domitilla corrects herself immediately. "Let's start over from the beginning. You first take the cat, Coaly. Then you go back and take the dog Cocodog. And then you leave Cocodog with Coaly… Oh no, that doesn't work either! Darn! There really is no solution!"

Do you agree with Domitilla? Do you also think no solution exists?

The seer seems truly sorry: she knows how important the Fair is for Drusilla; how hard she worked to attend it.

After a few moments, Drusilla breaks the silence, suddenly enthusiastic:

"But it's obvious! You just gave me an idea!"

"Really?" exclaims Domitilla, surprised.

"Sure, you helped me understand exactly where I was going wrong!" Drusilla approves with a grateful smile. "I think I've found the solution now!"

“What is it?” asks the seer, hesitant and incredulous.

“Listen carefully,” begins Drusilla. “Grey cannot stay alone with Coaly. And Coaly cannot stay alone with Cocodog. So first I must take Coaly. I leave him at the Fair and come back immediately. Clear so far?”

Domitilla nods mechanically, not sure where this is going.

“Then I take Grey and bring him to the Fair. Once there,” Drusilla continues, “I leave him there but take Coaly back, so that cat and mouse are not alone together.”

“You want to bring Coaly back home?”

“Exactly. I will bring him back home and then take Cocodog. Once I arrive at the Fair with Cocodog, I will leave him with Grey and return again to get Coaly. What do you think?”

Drusilla smiles triumphantly.

Domitilla looks thoughtful. But after a few moments, she exclaims enthusiastically, “You’re right: it works! Sure, you’ll have to make several back-and-forth trips, but in the end, they will all be together at the International Magic Fair. And no one will have gotten hurt!”

The young witch nods happily.

"Then I'll just have to set up my stall and have everyone try my potions," she concludes dreamily.

"You'll have to remember to buy a new wand," adds Domitilla with a smile. "A wand that can… withstand explosions!"

What It's About

Sometimes, a problem may seem impossible to solve, especially when, like Drusilla, we begin with a bit of discouragement and don't believe there could be a solution from the start. However, as we've seen, there is often a solution, even if it's hard to find.

If we think in the wrong way, we might end up convincing ourselves that no solution exists, even if someone tells us otherwise. In such cases, it's better to discuss it openly.

When we share our thinking with someone, we might realize on our own where our reasoning went wrong. Sometimes, it's others who point out where we've made a mistake. There's no shame in being wrong since everyone makes mistakes from time to time (even if they probably don't boast about it or tell us about it…). The saying "You learn by making

mistakes" is truly valuable! And by confronting others, we learn a lot.

But back to our story: how was Drusilla's problem solved? In this case, we must ***rule out all impossible options****, like leaving the cat and the dog together, or leaving the cat with the mouse. It becomes clear that the only animal that can be transported first is the cat.*

We can also make progress by thinking in a similar way, trying to avoid missteps and choices that would bring us back to where we started. For instance, we could take the cat to the Fair and immediately bring it back home, but that would be pointless.

There are different types of problems that can be solved like this, by trial and error or by considering the possible choices: one option may be absolutely ruled out, or, between two valid choices, one may be better than the other.

The problem becomes more complicated when there are many choices, and it's not immediately obvious which one is the best: for example, a chess player must only choose the best moves… and that is really difficult!

A LABEL FOR EVERY BOX

Can you figure out what's really inside three packages of muffins, knowing that all their labels are wrong and that you can only open one package and taste one muffin?

"It's paradise here!" exclaims Alice, overjoyed, unable to take her eyes off the pastry bag with which a young apprentice pastry chef is filling small puff pastry bases with cream.

The little girl's mouth is watering. Like her, her classmates are eyeing the dried or candied fruits, the

creams of various colours and flavours, the sugar decorations, and the cookies freshly taken out of the oven.

The students of the CM2 A class are visiting the laboratory of the oldest pastry shop in the city, “The Three Meringues.” It is Monsieur Pralin himself, the owner, who is serving as their guide: an older, patient man who has many stories and anecdotes to share.

Monsieur Pralin is the great-grandson of the founder of “The Three Meringues” and has become a pastry chef himself, like everyone in his family. Proudly, he explains to the children that the name of the pastry shop comes from a habit of his great-grandfather: every Sunday, he would prepare three personalized meringues for his three daughters. He also talks about how the tools used in pastry making have changed over time, becoming increasingly more technological.

“Even though,” he adds proudly, “no machine, no matter how advanced and precise it is, will ever replace the know-how and imagination of a true pastry chef!”

The children listen, thrilled, intoxicated by the smell of the cakes, enchanted by the sight of the brioches, tarts, and all sorts of pastries. Yes, Alice is

absolutely right: the laboratory of “The Three Meringues” is the place that comes closest to paradise!

The tour is almost over. Monsieur Pralin is about to say goodbye to the children when one of them, Matthew, timidly raises his hand.

“Monsieur Pralin…,” he begins hesitantly. “Before we leave, could I please taste a pastry?”

“Me too!” echoes Fabrice enthusiastically, rushing to add, “Just one… please!”

Monsieur Pralin smiles kindly. “Of course! In fact, even better,” he announces. “There, on the pastry counter, there are already prepared snack bags. One for each of you. And for each of your teachers, of course. But if you want them, you’ll have to solve a little riddle.”

The children react with astonishment and excitement, starting to shout with joy and impatience.

“Let’s see, children,” continues Monsieur Pralin. “How are you at solving riddles?”

In a disorganized chorus, the children’s voices mix and overlap with a series of:

“Very well!”

“Eh… so-so…”

“I love riddles!”

“So, so!”

“I’m a champion!”

“Oh no!!! I hate it!”

Monsieur Pralin laughs again. It’s clear that he’s having a lot of fun talking with the CM2 A class.

“So, do you want to try to solve the riddle I have prepared for you?” he asks, prompting an immediate and loud chorus of “Yes!”

“Good, follow me!” he adds, still laughing.

The pastry chef leads the children into an adjoining room to the laboratory, full of boxes of already prepared cakes, wrapping paper with the golden logo of “The Three Meringues,” and shiny ribbons ready to be used. He then starts rummaging through the contents of a shelf. The children, impatient and curious, hold their breath.

Suddenly, Monsieur Pralin turns around, holding a tray, which he places on a table right in front of the group of children. On the tray are three charming rectangular packages, about the size of the palm of a hand. The boxes are white, adorned with the unmistakable logo of the pastry shop, and are tied

with a beautiful red ribbon with a label. The children look at them carefully, with a hint of suspicion.

“Well,” begins Monsieur Pralin solemnly, “as you can see, here are three packages. In each of them, there are two muffins. The muffins can be either vanilla or lemon. But there’s a problem: all the labels on these packages are wrong. Someone a bit distracted mixed them up by mistake.”

“Oh! Who did that?” asks David worriedly, receiving nothing but a chorus of “Shhh!” “Quiet!” and “Be quiet!” in response.

Monsieur Pralin smiles.

“It doesn’t matter who did it,” he replies. “I’m sure you’ll sort everything out and figure out what’s really in each package. And if you succeed before noon,” he adds, glancing quickly at the clock on the wall, which shows 11:45, “I promise you that the snack bags will be yours.”

An “Oh!” of astonishment and excitement immediately spreads through the group of children, as their excitement grows.

“So, we have to guess what’s in the three packages?” asks Alice, looking at the table and the three elegant boxes on the tray.

“Exactly,” agrees Monsieur Pralin, “but you have to follow the rules I’m going to give you.”

The children immediately fall silent, ready to listen.

“As I said, there are two muffins in each box. They can be either lemon or vanilla, but they look identical. To know if it’s a lemon muffin or a vanilla muffin, you need to taste them,” explains the pastry chef. “You see, each box has a little label, but the inscription is wrong. The contents of each box don’t match what’s written on the label.”

The children nod, following the pastry chef’s words attentively.

“The three inscriptions are: ‘Vanilla & Vanilla,’ ‘Lemon & Lemon,’ ‘Vanilla & Lemon,’” continues Monsieur Pralin. “Your task is to figure out where the two vanilla muffins are, where the two lemon muffins are, and finally, which box has the vanilla and lemon mix. But to do this, you can only open one box and taste only one of the muffins inside.”

“Only one?!?” exclaims Victor, disappointed. “And how are we going to do that?”

“It’s doable, don’t worry! You just have to figure out how,” Monsieur Pralin immediately replies, giving him an encouraging smile. “If you really

want to win your reward, you'll have to solve this problem without the help of the adults here. Before noon. Is that clear?"

The children nod again, eager to start the challenge.

"Good! Good luck!" concludes the pastry chef, stepping aside and allowing the group to gather around the table.

"Okay," begins Matthew, observing the three boxes. "Let's open one and see what's inside. That way, we'll know what's in the others by elimination."

His classmates look at each other, hesitant.

"Wait," answers Stella. "Are we sure that if we open a box at random, we'll know what's in the others afterward?"

"Let's see…" murmurs David. "If we open the 'Vanilla & Vanilla' box, we'll be sure that the muffins inside aren't both vanilla because Monsieur Pralin told us that the labels were mixed up. So, we could either find 'Vanilla & Lemon' or 'Lemon & Lemon.'"

"That's true. But we can only taste one muffin…" thinks Alice. "How will we know for sure if it's 'Vanilla & Lemon' or 'Lemon & Lemon'?"

“Well… if we taste one muffin and find that it’s vanilla, then it’s good!” replies Fabrice, encouraged.

“We’ll know for sure that the box is the one with the mix. And we’ll recognize the others by elimination…”

“You’re right!” rejoices Victor. “Then we’ll only have the ‘Lemon & Lemon’ box and the ‘Vanilla & Vanilla’ box left, but we know for sure that each one contains identical muffins. However, the ‘Lemon & Lemon’ box can’t contain two lemon muffins because the label is wrong; so, it must contain two vanilla muffins…”

“Stop, wait a minute!” interrupts Matthew, sceptically. “And if, when we taste, we find a lemon muffin, what do we do? Should we try to guess?”

“If we find a lemon muffin…” Fabrice thinks hesitantly. “Yes, then we’ll still have some doubt…”

“Darn! So, that doesn’t work…” sighs David, disappointed. “What a shame…”

“That’s right, it doesn’t work that way…” replies Sophie. “But we shouldn’t give up: Monsieur Pralin said there’s a solution. And we still have a little time to find it, right?”

In your opinion, what strategy should be adopted to know for sure the content of the three packages?

The children stay silent for a few minutes. Then, Stella pulls out a small notebook and a pencil from her backpack.

“What are you doing?” asks Matthew, intrigued.

“I’m starting to get confused; I need a piece of paper and a pencil to note down the combinations,” she explains with a shy smile.

“Just before, our problem was the mixed box,” says Sophie, as if thinking out loud. “How will we know if the box is mixed or not if we can only taste one of the two muffins?”

The other children nod, looking thoughtful. Each of them is focused on solving the problem: the prospect of enjoying a delicious snack prepared by the famous patisserie “The Three Meringues” is very tempting...

Suddenly, Stella seems to jump.

“Guys, there’s a way to know for sure that the box we’re going to open isn’t mixed!” she exclaims.

“And what is it?” David asks.

“We know that all the labels are wrong,” Stella explains energetically.

“That’s right!” echoes Alice, smiling. “If the labels are wrong, then there can’t be two muffins with different flavours in the box with the ‘Vanilla & Lemon’ label, but there will definitely be either two lemon muffins or two vanilla muffins!”

A smile also spreads across Fabrice’s face.

“Yes, that’s true! If we open the box with the ‘Vanilla & Lemon’ label, we’ll know right away that both muffins are identical. If we try one, we’re 100% sure of the flavour of both! From there, we can also figure out what’s in the other boxes...”

“So,” Matthew summarises, “if we open the ‘Vanilla & Lemon’ box, taste one muffin, and it’s, let’s say, lemon, we’ll know that the correct label is actually ‘Lemon & Lemon.’ Then, only the ‘Vanilla & Vanilla’ and ‘Lemon & Lemon’ boxes will be left. Since the labels are wrong, the ‘Vanilla & Vanilla’ box can’t contain two vanilla muffins, and the one labelled ‘Lemon & Lemon’ can’t contain two lemon muffins. Anyway, we’ll have already figured out the lemon muffins.”

“So, by elimination, the ‘Vanilla & Vanilla’ box must be mixed, and the ‘Lemon & Lemon’ one

must, in fact, contain two vanilla muffins," concludes Victor triumphantly.

"Now, if, when we taste the muffin, we find that it's vanilla, then the box we opened is definitely 'Vanilla & Vanilla.' The rest of the reasoning is the same: just swap the words 'Vanilla' and 'Lemon'!" concludes Sophie, satisfied.

"What do you think? Shall we open the 'Vanilla & Lemon' box?" asks Fabrice with a mischievous smile.

"Yes!" the other children shout in unison.

"Can I taste it? Or at least have a little piece?" David asks, his mouth watering.

His friends burst into laughter, amused and satisfied. They're ready to put their strategy into action and enjoy a delicious happy ending.

And that's how the students of CM2 A finally get to taste the lovely snack prepared for them by the famous patisserie "The Three Meringues".

What It's About

In this riddle, one particular aspect of the children's thinking catches our attention: there are two different possibilities to consider, but the

students in class CM2 A focus on the first one and then convince themselves that the reasoning works the same way for the second one.

Well, mathematicians also often act this way to shorten and speed up the solution to a problem. For example, a mathematician would say: "Let's assume, without losing generality, that the muffin we tasted is lemon. If not, it doesn't change anything, ***just swap*** *vanilla with lemon, and the result will be the same."*

Fortunately, in our story, the problem is perfectly symmetrical, and the exchange can be made easily. In fact, when solving the riddle, it doesn't matter what flavours we are dealing with: the story and reasoning would work just as well with strawberry and raspberry flavours...

However, to avoid being misled and to understand each other clearly, we need to be really sure that the remaining possibilities, the ones not analysed in detail, are truly analogous. These little shortcuts can be used among mathematicians, but don't try to use them with your teacher! For example, if in an exercise you are asked to do 10 additions, it's not enough to calculate the first sum and then say that the others are done the same way…

What are riddles and puzzles for? To have fun, you might say, to test yourself, to learn, to train your mind... The good news is that you don't have to solve them all by yourself. Most riddles and puzzles are quite difficult when you have little experience. However, sometimes all it takes is a tiny hint to find the right track...

Often, when faced with a difficult riddle or puzzle, we tend to get discouraged and admit defeat. In fact, the best thing to do would be to talk to the person who gave you the riddle, ask for a hint, or get guided to find the solution (perhaps on your own). In any case, the most important thing is to understand the solution.

Learning mathematics works the same way: at first, you are guided and helped. Also, before solving mathematical problems on your own, it's useful to understand how others have solved similar problems: sometimes, you can adapt a strategy you've already seen, and things become much easier. So now that you've seen how to solve this riddle, you'll be able to solve similar ones by yourself. By the way! Some companies test people who want to work for them with logical riddles!

MERLIN’S MAGICAL BIRTHDAY

There are many people at Merlin the wizard's birthday party. They all greet each other with handshakes. Among all the guests, there are at least two who have shaken hands with exactly the same number of people. Can you explain why?

The twelve chimes of midnight echo in the dark and silent hall.

Suddenly, as if pushed by a gust of wind, the huge wooden door opens. Under the grand crystal

chandeliers, candles light up and start to glow, while joyful music fills the air.

Thick cloths that covered the long tables are suddenly whisked away, revealing all sorts of dishes and drinks: stews, tasty breads, cakes, cookies, and puddings. They rest on shining white tablecloths embroidered with gold.

In the blink of an eye, the quiet hall is filled with light and sound. Elegant and distinguished guests start arriving right on time at the entrance. The great party in honour of Merlin the wizard can finally begin!

At the stroke of midnight, the world's most famous wizard turned 850 years old! His friends, family, and fellow wizards came from all over the Earth to celebrate. It's a very private party, only for the most important people in the magical world. Everyone will talk about this party for days!

Fairies and elves, witches and wizards, enter the big hall decorated for the occasion, one after another.

Merlin, with his famous long white beard, his pointy hat, and a blue silk robe that touches the marble floor, waits at the entrance, greeting each

guest with a smile, a few kind words, and a handshake.

Next to him stands his young apprentice, Cedric, a smart and magically gifted boy, but still inexperienced.

Feeling proud, excited, and a bit awkward, Cedric watches the guests arrive. He bows respectfully and admiringly to each of them.

Among the guests are internationally famous characters whose names appear often in newspapers, magazines, and magical TV news. Still, the most famous of all, without a doubt, is Merlin himself, the star of the party.

While the guests fill the hall, Cedric watches them form small groups, talking and shaking hands. Sometimes the handshakes are friendly and warm, other times a bit colder and more formal. Some guests seem like old friends, others exchange wary looks, as if old rivalries divide them. But the overall mood is cheerful and relaxed. Everyone has come to celebrate Merlin, who watches happily from his place.

The hall is now crowded and buzzing with life and voices.

“Do you recognize anyone?” Merlin asks his young apprentice, leaning towards him.

Cedric nods eagerly, a bit dazzled by seeing so many famous faces. For a boy who hopes to join the circle of the world’s most admired wizard one day, being at Merlin’s birthday party feels like a dream.

“Do you want me to introduce you to some of your heroes?” Merlin asks, stroking his thick white beard.

Cedric’s eyes open wide.

“Yes, of course! Yes, sir!” he answers quickly, excited and impatient, and then adds almost begging: “Please!”

Merlin smiles. Behind his glasses, his eyes sparkle with pleasure.

“I will, I will,” he reassures him. “But on one condition.”

“A condition?” repeats Cedric, surprised and worried.

He knows his master can be strict and demanding…

“On the condition that you answer one question. Just one,” says Merlin, fixing his grey eyes on the

boy's blue eyes. "Remember, rewards must be earned!"

Cedric nods, looking serious and attentive.

"And what is this question?" he asks nervously.

"Listen carefully," Merlin begins, speaking each word slowly, like a teacher. "At a party like this, there are always at least two people who have shaken hands with exactly the same number of guests. Can you tell me why?"

The boy looks at him, a bit confused.

"You're saying you are absolutely sure that here, among all these guests, there are at least two people who shook hands with the same number of people? And you know this without counting?" he asks, puzzled.

"Yes, I can be certain," Merlin replies. "And there may even be more than two people like that…"

Then, after a brief pause, Merlin adds:

"Remember that no one can shake their own hand, and that we only count the first time two people shake hands (not if they do it again). How can I be sure of what I say?"

And you, can you answer Merlin's question?

Cedric lowers his head, thinking hard. He looks around as if he could find the answer in the bustling crowd. Witches, wizards, elves, and fairies fill the hall, chatting, laughing, tasting stews and treats, and sipping drinks. How can you be sure who shook hands with whom?

The young boy thinks and thinks again. At a party, someone might shake hands with everyone, and someone else might shake hands with only a few people. Maybe there is someone who shook hands with no one at all. How can we be sure that at least two guests have shaken the same number of hands?

“You won’t find the answer by looking around,” Merlin says kindly. “You already have all the information you need. Everything you need is right here in your head!”

Saying this, he taps a finger on Cedric’s forehead. Cedric looks at him, wondering.

“Really?” he says, surprised. “This seems so complicated…”

Merlin smiles.

“Sometimes, to solve what seems like a very abstract problem, it helps to imagine a simpler,

concrete example," he suggests gently. "Think about numbers and start from there."

Cedric listens carefully.

"I can imagine a party with fewer guests…" he murmurs, hesitating.

The wizard nods, encouraging him to continue. "I imagine a party with 10 people," Cedric says, thinking aloud. "If there are 10 people, how can I be sure that two of them have shaken hands with the same number of people?"

Deep in thought, Cedric narrows his eyes.

"Maybe you should consider the smallest and largest number of people someone could have greeted…" Merlin suggests, still talking calmly.

Cedric nods.

"There could be someone who greeted everyone else," he says after a moment of silence. "So, if there are 10 people total, at least one might have shaken hands with all 9 others.

But," he adds at once, "there might also be someone who shook hands with no one. In that case, it's impossible for someone else to have shaken hands with everyone, because not everyone got a handshake from that person who didn't shake any hands."

“Well done. As you see, the two possibilities you mentioned cannot both happen at once,” notes Merlin.

“Yes,” Cedric agrees. “So, if everyone shook at least one person’s hand, the number of handshakes per person goes from at least 1 up to 9 at most. But if there is someone who shook no hands, then the number of handshakes per person goes from 0 at least up to 8 at most. Because in that second case, no one could have shaken hands with everyone.”

Merlin smiles, satisfied.

“Well done, young man!” he says. “Now there’s just one last step. You have 10 guests at the party and a possible number of handshakes that goes from either 1 to 9, or 0 to 8.”

Cedric nods again, more confidently this time.

“I’ve got it! I understand!” he exclaims. “There are 9 possible numbers of handshakes per person (either from 1 to 9, or from 0 to 8). But there are 10 guests. If I must choose 10 times from only 9 possible numbers, I’m forced to repeat at least one number! That means at least two people share the same handshake count!”

The young boy smiles brightly.

“So, among the 10 guests at the party, at least two shook hands with the same number of people,” he concludes, his eyes shining with satisfaction. “This reasoning would still work even if there were more than 10 guests…”

“Exactly!” Merlin congratulates him with a gentle pat on the shoulder. “No matter how many guests there are at the party, this reasoning always works. Or more simply, as long as there are at least two people at the party—which is always true for a real party—this must happen. Now… which magical stars would you like me to introduce you to, Cedric?”

What It’s About

*Merlin’s question to Cedric is inspired by the “**pigeonhole principle**,” a classic idea in mathematics about repetition. In simple terms, if you have more items than boxes, at least one box must contain more than one item. This guarantees repetition.*

For example, imagine you have a box with socks of only four different colours. If you pick five socks at random, you are sure to have at least two socks of the same colour (even though you don’t know which

colour they will be). You might even get more than two socks of the same colour, or maybe two pairs of different colours.

A more precise version of the pigeonhole principle also tells us how many times you get the same outcome.

For example, if you flip a coin five times, you will get at least 3 of the same result (like at least 3 heads or at least 3 tails). How do we get this number? We divide 5 by the number of different outcomes (2: heads or tails). 5 divided by 2 is 2.5, not an integer. So, we take the next whole number, which is 3. That means no matter what, among your 5 flips, at least 3 will be the same result.

To understand this calculation, it's important to think that the smallest number of identical results happens when the outcomes are as evenly spread out as possible.

For example, when tossing a coin four times, the ***worst luck*** *would be getting two heads and two tails. Now, if we toss the coin a fifth time, we already know that it will land on either heads or tails. So, after five tosses, we will have at least three heads or at least three tails. This means that 3 is the* ***best number*** *we can reach, no matter what, after five tosses.*

Note that often, mathematicians focus on the worst case to understand a problem. This is because the other cases we might consider can usually be solved in a similar, or even easier, way.

In the story, we also learned that to understand and solve a problem, it's helpful to illustrate it with an example — meaning we use a real-life example to help explain.

Luckily, many math concepts can be understood through examples: when the reasoning and steps used to understand and solve specific cases lead us to understand and solve the general case.

But be careful! To prove a general rule, it’s not enough to just take an example. You need to be able to figure out a general rule from that example and make sure (or convince yourself without any doubt) that it works every time.

THE MIXED-UP FRAMES

Can we put a series of paintings back into the correct order by swapping only two at a time?

Finally, the big day has arrived: it's the opening day of Picto's very first art exhibition! The King and Queen of Castlevalley allowed him to show his works in a beautiful corridor of the Sparkling-Rock Castle. The exhibition will last ten days. If people like his artwork, Picto will receive the prestigious title of "Court Painter."

The young artist is naturally nervous and excited. This is the first time in the kingdom's history that a dragon—and yes, Picto is a dragon (although a small one)—has a chance to get such an important job at the court.

What an honour it would be! His whole family would be proud. The young artist is overjoyed!

The corridor of the exhibition has a very long red carpet adding a nice colourful touch. On one side, there are large windows offering a breathtaking view of Castlevalley. Picto spent the entire morning hanging his paintings on the opposite white, empty wall. After much thinking and many hesitations, he decided to show a series of paintings called "From A to Z." It includes all the letters of the alphabet. Each letter is in a small frame, and each frame is also part of a story told in pictures. Because of this, it's important that the letters are arranged in alphabetical order, from A to Z, exactly as the title suggests.

"Are you ready? In one hour, the king and queen will order the doors to open!" cries a tiny flying fairy, landing on the dragon's shoulder.

It's Flash, Picto's friend. The fairy and the dragon grew up together in the Castlevalley woods and have played together since they were small. It was Flash

who first noticed Picto's artistic talent and encouraged him to show his work to the rulers.

"Yes," answers Picto, looking with satisfaction at the paintings on the wall. "Everything seems in order… except my stomach!"

As he says this, he rubs his belly.

"You're nervous, right?" asks Flash, understandingly. "You know what you should do? Go to the kitchen, have a nice cup of tea, then relax on a comfy sofa. That way, when the doors open, you'll feel great and ready to greet the visitors!"

Picto doesn't seem convinced.

"I don't want to leave my paintings…" he murmurs, shaking his head slightly.

"Oh come on, what could possibly happen to them?" teases Flash, laughing. But seeing how serious Picto looks, he tries to reassure him: "Don't worry, I'll stay here and keep an eye on them while you go have something to drink and rest. I promise, your paintings are safe with me!"

With a solemn gesture, the fairy places a hand on his heart. After a few moments of hesitation, Picto sighs.

“You’re right… I really need to relax,” he murmurs, as if talking to himself. “And since you’re staying here… I know I can trust you!”

Flash nods, smiling, and starts fluttering around the corridor again.

“From up here, I can watch everything!” he exclaims. “Go on, take a break!”

Picto gives him a grateful smile and crosses the corridor toward the castle kitchens. The fairy keeps flying around happily.

After a few moments of silence, a ball rolls loudly along the red carpet, followed by two curly-haired children. They are Prince Astre and Princess Lune, running breathlessly through the corridor, causing quite a racket!

“Hey, kids! Hey!” calls the fairy in a strong voice, flying down toward the floor. “You can’t play ball here! There’s an exhibition, and visitors will arrive soon!”

The two children stop, panting, and turn to look at him.

“It’s not our fault…” says Prince Astre with a mischievous smile. “The ball got away.”

“You know, it’s a rather naughty ball…” adds Princess Lune with a small giggle.

The fairy smiles back, charmed by their cute faces.

“Alright, alright,” he replies.

Then, picking up the ball from the floor, he adds: “Come on, let’s take this tricky ball away from here. Forward, march!”

With these words, he leaves the corridor, flying toward the castle courtyard, followed by the two children. The echo of their footsteps and voices fades away.

Now the corridor is deserted and silent again. But suddenly, a sneaky shadow appears on the red carpet. Quiet but fast, it moves back and forth in the room. Oh no! It’s taking the paintings down and re-hanging them in different places on the wall!

“Hey! What are you doing? Stop!” suddenly shouts Flash, who has just returned, flying quickly into the corridor. “Don’t touch those paintings!”

The shadow is startled and, after hanging the last frame, prepares to run away quickly.

“Carbo, what have you done?” cries the fairy, horrified to see all the frames mixed up, and recognizing the culprit.

It’s Carbo, who used to be the court painter but was fired by the king because of a failed painting and sent to work in the stables.

“This exhibition will be a disaster!” Carbo exclaims with a mocking smile. “The king will notice, kick out the dragon, and I’ll get my old job back!”

“You know very well it’s not Picto’s fault you lost your job!” shouts Flash, angry and upset. “I’ll tell everyone you tried to sabotage the exhibition!”

Carbo laughs.

“And I’ll say it’s not true,” he replies. “You have no proof against me! It’ll be your word against mine. People will think you’re accusing an innocent man just to help your talentless friend…”

Then, throwing a defiant look at the fairy, Carbo leaves the corridor in a hurry.

Flash, feeling sorry, looks around: The exhibition will start in about half an hour, and all the paintings are out of order! What will Picto say when he sees this mess? How will he react? His heart will break! He’ll never trust his best friend again. Even worse—Flash trembles at the thought—he might not want to be friends anymore…

The poor fairy cannot let that happen. He must find a solution, fast. But alas, even though the frames are small, they're still too heavy for a tiny creature like him to move alone. What to do?

At that moment, an idea comes to him. In the blink of an eye, he flies through the corridor to who knows where. A few minutes later, he returns with Prince Astre and Princess Lune following behind, looking curious and a bit wary.

"I don't understand… What must we do?" asks the princess, looking around. "All the paintings are still hanging on the wall, just like before…"

"But they're not in the right order," Flash explains seriously. "They've been moved. We have very little time to put them back."

"And what's the right order?" asks the prince.

"On each frame, there's a letter of the alphabet. We must arrange them from A to Z, starting from the left," says the fairy, pointing to one end of the corridor, the side closest to the entrance.

The children nod. They care a lot about Picto and are ready to do anything to help him.

"Okay… Where do we start?" ask Astre and Lune, looking around.

“Good question…” murmurs the fairy, deep in thought.

“We could take them all down and place them one by one…” suggests Astre, glancing around.

“I’m not sure… I’m afraid we’d make a big mess,” answers Lune, worried. “We can’t put all the paintings on the floor—there’s no room, and we might trip over them. Leaning them against the wall is risky too—we could accidentally kick them while trying to hang another frame. We must not damage Picto’s paintings!”

What do you think? What strategy could Flash, Astre, and Lune use to put the paintings back in the right order?

Astre groans, annoyed. Then he walks over to the letter A, which ended up in the middle of the wall, far from where it should be.

“In the meantime, I’ll take this A and put it at the beginning!” he exclaims. “We have to start somewhere…”

Removing the letter A, he takes it to where it should be, at the start of the alphabet, as the first

painting. Right now, in that spot, is the letter G… what confusion!

“Wait, I’ll take down the G so you have room,” says Lune. “Where does G go?”

The two children look around, then start counting letters.

“Wait,” interrupts Flash suddenly. “No need to figure that out now. Put G in the empty space where A used to be!”

The two children look at him, puzzled.

“But that’s not its correct spot!” they protest in unison.

“Do as I say!” insists the fairy, flying around them. “Then we’ll get B and put it next to A, where P is now. We’ll move P to the empty spot where B was before. Then we get C, and so on…”

“So… you’re suggesting we swap two paintings at a time?” Lune asks, hesitantly.

The fairy nods.

“You think that will work? You think we’ll be finished on time?” the little girl asks again.

“Yes, yes, trust me,” Flash reassures them. “If we do it in an organized way, we won’t get confused or trip over the paintings.”

“What are we waiting for? Let’s get to work!” exclaims Astre energetically.

Following Flash’s instructions carefully, the children run back and forth along the corridor. They work like a well-organized team, while the fairy watches anxiously.

Letter after letter, always swapping two at a time, they manage to place the letter Y correctly to the right of the letter X. At this point, all letters from A to Y are in their correct places. To the right of Y, there’s only one letter left: Z, the last letter of the alphabet, already in the correct spot!

“Hooray! We did it!” cheer Lune, Astre, and Flash together.

The fairy sighs with relief: Picto’s exhibition is safe, and so is their friendship!

What It’s About

This story is based on ***the mathematical theory of permutations****, which are ways to rearrange objects. There are many possible permutations, even with a small number of items.*

For example, how many ways can you arrange 10 people in a line? It’s the product of the numbers

from 1 to 10. That's more than 3 million possibilities!

Arranging the 26 letters of the alphabet has a huge number of possibilities— a 27-digit number!

How do we get these numbers? Let's take a simple example: we want to arrange 3 people in a line. For the first position, we have 3 choices. Once we pick someone, 2 people remain for the second position. After that, the last person goes in the third position. In total, 3 × 2 × 1 = 6 possible arrangements.

*A key idea in permutation theory is that any arrangement can be made by a series of swaps, or "**transpositions**," each swapping just two objects at a time. Also, each transposition can be achieved by swapping neighbouring items step by step.*

For example, if we want to exchange A and D in the alphabet: first swap A with B, then A with C, and finally A with D. After that, swap D back with C, then D with B, until we get the exact order we want.

Another interesting point is that objects can move in groups called "cycles." For example, if the teacher rearranges the entire class, Anna takes Beatrice's seat, Beatrice takes Claude's seat, and

Claude takes Anna's seat. This is a cycle among Anna, Beatrice, and Claude.

*We also talk about the "**parity**" of permutations. A permutation is "even" if it can be obtained by an even number of two-object swaps, and "odd" if it needs an odd number of such swaps.*

You might know the "15 puzzle," a classic sliding puzzle with 15 numbered tiles on a 4x4 grid with one empty space. Keeping the empty space in a certain corner, only half of all possible arrangements are actually reachable, because you can only get even permutations (or only odd, depending on the rules).

THE TWO DICE

If you roll two regular 6-sided dice and add the numbers that appear on top, which total is the most likely?

"Pfff!" sighs Eddy sadly, dropping his pencil on his notebook. "We'll never finish reviewing everything for tomorrow…"

Next to him, Laura sighs too, feeling discouraged.

"There are still so many topics to review…"

The two children are in 4th grade, studying together for tomorrow's math and geometry test. But the more time passes, the lower their spirits sink.

"Let's see how many pages are left," says Laura, taking the book from Eddy's hands and flipping through it.

"Way too many!" Eddy replies. "The test covers almost the entire year's program… How are we going to get through it all?"

"Let's just keep going and see how far we get," suggests Laura, tired, letting the open book rest on the table.

The page is colourful, full of numbers, signs, and drawings. At the top left, in big, bold, bright orange letters, is the title "Properties of Division." To the right of the title, there's a small drawing of a gnome with a white beard, a green tunic, and a bright red pointy hat. On his open palm are drawn the two dots of division, also in bold orange. The gnome seems to wink at the reader. His name is Mathmax, and he's clearly the textbook's little mascot.

"They should invent a way to remember everything after reading it once…" dreams Eddy. "That would save us a lot of time!"

Laura smiles.

“Or learn how to do one kind of operation once and then be able to solve all similar operations without effort or mistakes…” continues Eddy, daydreaming. “That would be much simpler.”

“Oh yes,” agrees Laura. “It would be so nice if that technique existed. Too bad it doesn’t…”

“Maybe it does, but nobody taught us,” Eddy suggests.

“If it existed, it would be famous… everyone would want to know it,” Laura objects. “If no one mentions it, it’s probably because it doesn’t exist.”

“Or maybe it exists, but it’s a secret!” whispers a tiny voice.

Both children jump, exchanging worried glances.

“Who spoke?” asks Laura, a bit frightened. “Was it you?”

Eddy shakes his head, looking around. They’re in his family’s living room, and nobody else is there. The door to the hallway is open, but no one is visible.

A soft, light laugh is heard.

Annoyed, Eddy stands up and looks behind the door, thinking one of his brothers is playing a prank. But there’s no one there. The hallway is empty.

“Eddy, Eddy…” Laura has jumped up too, eyes wide, pointing to a spot on the table.

“What the…” Eddy doesn’t finish his sentence because a shiver runs down his spine, stopping him.

He sees it too. There, on the table: something extraordinarily strange… A tiny little man, with a round, smiling face and a long white beard. He also has a pointy hat and a green tunic.

“Math-max?” stammer both children, astonished, recognizing their textbook’s mascot. “Is that really you?”

“In person!” exclaims the gnome in his small voice. “But please, call me Max. I don’t like fancy stuff.”

With these words, he stands on the open page of the book and starts hopping energetically.

“I couldn’t stand staying in that position without moving,” he explains. “With my hand outstretched in front of me. It’s very uncomfortable to stay like that all day, you know? Now I can finally stretch a bit.”

Next to the title, on the page about division’s non-commutative property, only the two bright orange dots remain. The gnome that held them has vanished

from the picture. Or rather, he left the page to jump onto the open book in real life.

“You’re real?” asks Eddy, approaching carefully and observing the strange little bearded man.

“Do I look fake?” replies the gnome, offended, still hopping.

The two children exchange cautious, amazed looks. Then slowly, they sit back down, right in front of Max.

“You live in the book?” asks Laura, both incredulous and curious.

The gnome nods.

“You can’t imagine how boring it is…” he says, sighing. “Staying still all day, listening to your complaints! No one ever laughs or has fun while looking at me! Yet I’m a really nice guy… very nice indeed!”

Eddy and Laura glance at each other: they’ve never opened their textbook smiling either.

Max seems to guess their thoughts.

“I know a foolproof way to study and learn quickly,” he exclaims, his face lighting up with a smile. “And I can teach it to you! But only if you play a little game with me…”

“Play with you?” repeats Eddy. “What does that mean?”

“It means that if you want my help, you must beat me at a game I choose.”

“At a game you choose?” repeats Laura. “What kind of game is it?”

With a snap of his fingers, Max makes a board game appear on the table, full of colourful squares. It looks like a giant game of “Goose.” In one corner of the board, there are two round tokens, one red and one blue, about the size of a cookie, and two normal-looking dice, white with black dots from 1 to 6.

The children watch, open-mouthed.

“This one is mine!” exclaims the gnome, snapping his fingers again and moving the red token. The token flies, spinning, and lands on a yellow square in the middle of the board.

“Wow! You’re really a magician!” exclaim Laura and Eddy together, watching with a mix of surprise and curiosity.

They wonder what else Max can do…

“Now, the rules!” announces the gnome seriously, clearing his throat. “As you see, my red token is on the board. On the other side are two dice and a blue

token. The game is: You must choose how many squares away from the red token you will place the blue token. Then you roll the dice and add their results, giving you a number from 2 to 12. If the number you get matches the distance you chose, you win. If you win, I'll be honoured to become your study tutor and help you ace tomorrow's test!"

"It's true, these are two 6-sided dice. So we must pick a number between 2 and 12," comments Eddy, scratching his chin.

"Yes," agrees Laura, also getting into the spirit. "But we must not choose just any number at random, right Max? We must find the number that's most likely to appear."

"Well, if you want the best chance of winning… you must think carefully!" says the gnome kindly. "Go ahead! Show me your reasoning!"

If you were in Eddy and Laura's place, where would you put your token?

"Okay…" murmurs Laura thoughtfully, nervously twirling her pencil between her thumb and index finger.

“Let’s start from the beginning,” thinks Eddy. “2 is not very likely because you only get it if both dice show 1. The same goes for 12, which you get only if both dice show 6.”

Laura nods approvingly, writing combinations on a yellow sticky note.

“3 is not very likely either, since you get it only by 1 and 2 or 2 and 1. But it’s still more likely than 2 or 12, because it can happen in two ways (1+2 and 2+1),” Eddy continues.

“You’re right. Now let’s look at 4,” says Laura. “You can get it with 2+2 or with 1+3 or 3+1. For 5, you can get 1+4, 4+1, 2+3, or 3+2.”

“Now for 6,” adds Eddy. “You can get it from 3+3, 5+1, 1+5, 2+4, or 4+2. And 7 you can get from 1+6, 6+1, 2+5, 5+2, 3+4, 4+3.”

The boy pauses and exclaims: “Wow! That’s a lot of combinations!”

Laura, focused, nods.

“It’s true, but let’s also check the remaining numbers,” she says softly. “For 8, we have 4+4, 2+6, 6+2, 3+5, 5+3.”

“Then 9 is 3+6, 6+3, 4+5, 5+4,” Eddy continues. “That’s fewer combinations than 6, 7, or 8.”

“10 and 11 have even fewer combinations,” thinks Laura after a short pause. “10 is from 5+5, 6+4, or 4+6. 11 is just 5+6 or 6+5.”

She notes down the last combinations.

“So the numbers that are most likely are 6, 7, and 8, because they have the most possible combinations,” concludes Eddy with a sigh. “Which one should we choose?”

Laura is silent for a moment, studying her sticky note. Then she smiles:

“Actually, one number is more likely than the others!” she says, showing the note. “6 and 8 each have a pair with identical numbers (3+3 or 4+4), but 7 can only be made by different pairs of numbers: 1+6, 6+1, 2+5, 5+2, 3+4, and 4+3. That’s six ways total!”

“You’re right!” exclaims Eddy. “7 is the most likely number! To have the greatest chance of winning, we must place our token seven squares away from Max’s token.”

“Exactly!” Laura nods.

“Are you really sure?” asks the gnome, who has been watching them silently all this time.

“Yes!” the children reply confidently.

“7 is definitely the most likely number,” Laura concludes. “Of course, we still need some luck…”

“Very well, now you can roll the dice,” says Max, magically moving the blue token seven squares away from the red one. “Good luck!”

As the children get ready to roll the dice, the gnome adds with a big smile:

“No matter what number comes up, I think I’ll help you study for tomorrow’s test anyway. You’re clever kids, and I like you a lot!”

What It’s About

*One **property of division** is that the quotient (the result of a division) doesn’t change if we multiply both the dividend (the number being divided) and the divisor (the number we’re dividing by) by the same number. For example, 10 ÷ 2 is the same as 20 ÷ 4. If you share 10 candies among 2 children, each gets 5. If you have 20 candies and 4 children, each still gets 5, and the same logic applies to 100 candies and 20 children.*

When you get older, you’ll need to remember and apply this property in everyday situations. You don’t need to memorize it word for word; you can always

look it up. What's important is understanding how to use it.

The math question about dice probability in this story is very useful in some board games like backgammon. Even though winning depends partly on luck, you can "help" luck by thinking carefully and choosing moves that increase your chances of winning.

When we roll two standard dice (imagine one red and one blue), each die shows a number from 1 to 6. There are 36 possible outcomes since each die can be any of 6 numbers, and 6×6=36.

If the dice are fair, each of these 36 outcomes is equally likely. But the sum of the two dice results can be from 2 to 12, giving us 11 possible sums. They are not all equally likely.

As Eddy and Laura discover, 12 can only happen one way: 6+6. The number 7 can happen six ways, making it the most likely sum.

So, we have 36 total outcomes, and some sums appear more often in these outcomes than others. That's why some numbers, like 7, are more likely.

*Mathematically, the sums form what's called a "**multiset**" of 36 elements. A "multiset" is like a set,*

but elements can repeat. ***The number of times*** *each element appears is important.*

Multiset theory, which is an extension of set theory, is both simple and interesting.

For example, you can combine two multisets by putting all their elements together, like pouring the contents of two pots into one bigger pot.

Or you might look for the smallest multiset that contains two given multisets. This might happen if you can't decide between two recipes and you consider the sets of their ingredients. If one recipe needs 6 eggs and the other needs 4 eggs, you don't need to buy 10 eggs total. You only need 6, because 6 eggs cover both cases.

Bravo! You have reached the end of this book.
All that's left is to wish you good luck in your next math adventures!

Sources and Essential References

In the future, we plan to gather useful links to help you explore the math topics discussed in this book more deeply, as well as to discover new ones that are also easy for young readers to understand. To stay updated, check this webpage from time to time:

https://www.antonellaperucca.net/histoiresfantastiques.html

A Cake for Two

This puzzle is inspired by game theory. It's the simplest type of "cake-cutting problem," known in English as *Fair Cake-Cutting*.

Escaping the Labyrinth

This puzzle is based on a result from the mathematical theory of labyrinths.

The Goblin Who Lies

This riddle is taken from the movie *Labyrinth* (1986).

The Curious Messenger

This story is based on a well-known element of cryptography related to the key-exchange problem, known in English as the *Key-Exchange Problem*.

Tea in the Library

This story is inspired by a classic math exercise. Problems of this type (with numbers that can be easily changed) are sometimes called the *Hourglass Problem*.

The Scale of Friendship

This story is centred on a classic math problem with many variations, known in English as *the Weighing Puzzle* or *the False Coin Problem.*

The Two Rabbits

The problem posed by Mrs. Tortoise is an adaptation of the *Bronx versus Brooklyn riddle,* taken from the book *My Best Mathematical and Logic Puzzle* (1994) by the American math populariser *Martin Gardner.*

Traveling with Cat, Dog and Mouse

This puzzle is a reinterpretation of the classic problem of the wolf, the goat, and the cabbages. Its oldest known form is found in a 9th-century manuscript called *Propositiones ad acuendas juvenes,* which is the oldest known collection of math games and riddles. It's sometimes attributed to the Anglo-Saxon philosopher and theologian *Alcuin of York* (735–804).

A Label for Every Box

This puzzle is adapted from the *Apples and Oranges riddle*, used in job interviews at high-tech companies and mentioned in *William Poundstone's* book *How Would You Move Mount Fuji?* (Little Brown & Co., 2003).

Merlin's Magical Birthday

This story is inspired by a classic problem based on the *pigeonhole principle.* It's sometimes called the *Handshake Problem.*

The Mixed-Up Frames

This story is based on some basic ideas of permutation theory.

The Two Dice

Max's question is inspired by some basic notions of *probability theory.*

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