The Hardest Logic Puzzle Ever

A warm-up logic riddle

In the movie Labyrinth the main character Sarah has to solve a logic riddle. She is facing two doors, one leading to a castle and one leading to certain death. There are two guards, and she can ask one yes-no question to one of them. Sarah also knows that one of the guards always tells the truth and the other always lies, but doesn't know who is the truth-teller and who is the liar. What question can she ask to know which door she has to open?
Here is the question that Sarah asks: "Would he [pointing at the opposite guard] tell me that this door leads to the castle?" In both cases the answer that Sarah gets is a lie: the truth-teller has to relate the lie that the opposite guard would tell, while the liar would negate the truth that the opposite guard would tell. So if the answer is 'yes' the door leads to certain death, and if the answer is 'no' the door leads to the castle.

The Hardest Logic Puzzle Ever

The so-called "Hardest Logic Puzzle Ever" is the following riddle [1].

> Three gods A, B, and C are called, in no particular order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language, in which the words for yes and no are da and ja, in some order. You do not know which word means which.

Some clarifications [3]: a single god may be asked more than one question; the questions and to which god they are asked may depend on the answers to earlier questions. Random acts as either a truth-teller or a liar. You can imagine that he flips a fair coin in his head: if the coin comes down heads, he speaks truly; if tails, falsely. Finally, the gods are very intelligent beings, and can understand even very complicated logical questions in English. Moreover, they know all their identities, plus common knowledge such as $1 + 1 = 2$.

Notice that the riddle is very complicated mostly because of the presence of the Random god (whose answers convey no information), and because of the language barrier.

The riddle without the language barrier

For the moment we suppose that we can understand the gods's language, so that – by translating – the answers to our questions are either yes or no. We start with some preliminary remarks.

- If you know that a god is either True or False (in other words, which is not Random), how can you determine his identity? This is very easy. You ask him a question for which you know the answer already, and you check if he tells the

truth or not. One possible question is "Is 1+1=2?". True would answer yes, and False would answer no.

- If you know that a god is True, you can determine the identity of the other two gods by asking him (pointing at one of the other two gods) "Is this god Random?" If the answer is yes, the god is indeed Random and the remaining god is False. If the answer is no, the god is False and the remaining god is Random.
- If you know that a god is False, you can determine the identity of the other two gods by asking him (pointing at one of the other two gods) "Is this god Random?" If the answer is no, the god is indeed Random and the remaining god is True. If the answer is yes, the god is True and the remaining god is Random.

By the above, it suffices to ask one first question to find a god which is not Random. Indeed, we can address that god our second question "Is 1+1=2?" to determine whether he is True or False. Then we inquire again by him whether "Is this god Random?", by pointing at one of the other two gods.

The question to find a god which is not Random is slightly more complicated. We can ask any of the gods, pointing towards one of the two other gods: "Are you True if and only if this god is Random?" The 'if and only if' is a logical way to put together two assertions: the global assertion is true either when both assertions are true or when both assertions are false. Let us analyse the possible answers. According to the answers we are going to determine one god which is not Random.

- If we are asking True, then an affirmative answer confirms that we are pointing at Random, while a negative answer means that we are pointing at False.
- If we are asking False (which lies), then an affirmative answer confirms that we are pointing at Random, while a negative answer means that we are pointing at True.
- If we are asking Random, then the answer does not carry any information. However, the god we are pointing at is not Random, and the same holds for the third god (neither the one we are interrogating, neither the one we are pointing at).

In any case, if we receive an affirmative answer, then the third god (neither the one we are interrogating, neither the one we are pointing at) is surely not Random. If we receive a negative answer, then the god we are pointing at is surely not Random.

The general riddle

Now we consider the true riddle in which the gods answer da and ja and we have no clue what that means. We keep the same strategy as above, by varying the questions a bit. Namely, we start each question by adding "The word da means yes if and only if ...". The dots stand for the questions as above, and in any case the dots stands for an assertion.

- If da means yes, then we get an answer 'da' if the god wanted to say yes and 'ja' if the god wanted to say no when asked about the truthfulness of the assertion.
- If da means no, then we get an answer 'da' if the god wanted to say yes and 'ja' if the god wanted to say no when asked about the truthfulness of the assertion.

In this way, independently on the meaning of 'da' and ja', we know how to interpret the gods' answers. With this adaptation of the questions we can solve the general riddle. Notice that this solution is basically the one given in [2].

For variations of the riddle we direct the reader to the English Wikipedia page [3].

References

[1] George Boolos, *The Hardest Logic Puzzle Ever*. The Harvard Review of Philosophy, Volume 6 (1996), pp.62-65 https://doi.org/10.5840/harvardreview1996615.

[2] T.S. Roberts, *Some Thoughts About The Hardest Logic Puzzle Ever*. In: Journal of Philosophical Logic, 30:609–612(4), December 2001.

[3] Wikipedia contributors, "The Hardest Logic Puzzle Ever," *Wikipedia, The Free Encyclopedia*, `https://en.wikipedia.org/w/index.php?title=The_Hardest_Logic_Puzzle_Ever&oldid=906163668` (accessed August 15, 2019).