Sum of the first natural numbers

\[\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\]
Square of a binomial

\[(a + b)^2 = a^2 + b^2 + 2ab\]
The area of the circle is $\pi \cdot r^2$
Sum of the first cubes

\[ 1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2 \]
Viviani’s theorem

“In an equilateral triangle the sum of the distances from any interior point to the three sides is equal to the altitude of the triangle”
**Sum of squares of Fibonacci numbers**

Fibonacci numbers: \( F_1 = F_2 = 1, \quad F_n = F_{n-1} + F_{n-2} \)

\[ F_1^2 + F_2^2 + \ldots + F_n^2 = F_n F_{n+1} \]
Sum of the first odd numbers

\[ 1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2 \]
Pythagoras’ theorem

\[ a^2 + b^2 = c^2 \]
“The area of the square inscribed in the semicircle is \( \frac{2}{5} \) times the area of the square inscribed in the circle.”
Geometric series of common ratio $\frac{1}{4}$

\[
\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots + \frac{1}{4^n} + \ldots = \frac{1}{3}
\]
\( \pi \) is between 3 and 4

\[ 6 < 2\pi \]

\[ \pi < 2 \cdot 2 \]

\[ 3 < \pi < 4 \]
Geometric series of common ratio $\frac{1}{2}$

\[
\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n} + \ldots = 1
\]
Arithmetic mean and geometric mean

\[ a : G = G : b \]

\[ G = \sqrt{ab} \quad M = \frac{a+b}{2} \]

\[ \sqrt{ab} \leq \frac{a + b}{2} \]
Pentagonal and triangular numbers

\[ P_n = 3T_{n-1} + n \]