Unmasking Recurrence Sequences

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There are many interesting sequences of numbers that can be described by the first values ('initial terms') and by a rule ('recurrence').
The **Fibonacci sequence**

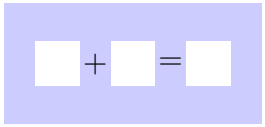
$$0,\ 1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144,\ \ldots$$

starts with the numbers 0 and 1. All other terms can be obtained with this simple rule:

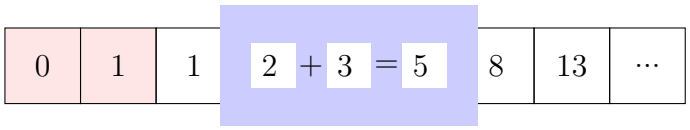
Add the two previous terms.

Indeed, we have $0+1=1$, then $1+1=2$, then $1+2=3$, then $2+3=5$, and so on.

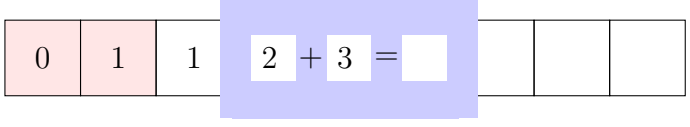
The sequence is a stripe of numbers, and the recurrence is a mask!



The recurrence relates neighbouring terms in a prescribed way: **You always see a true equality when the mask goes on the stripe!**

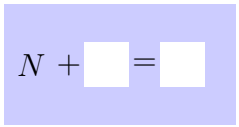


The recurrence lets you deduce all terms from the initial ones: **You can use the mask to compute the next term!**

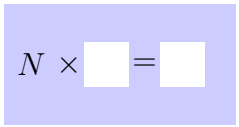


Periodic sequences (for example the sequence $0,1,1,0,1,1,0,1,1,0,1,1,\ldots$) regularly repeat finitely many values. The mask looks like this:

Arithmetic sequences (for example the sequence $1,3,5,7,9,11,\ldots$) require you to add one same number N at each step. The mask looks like this:

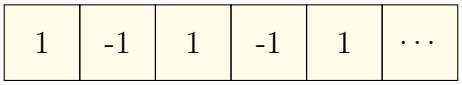


Geometric sequences (for example the sequence $1,2,4,8,16,32,\ldots$) require you to multiply by one same number N at each step. The mask looks like this:



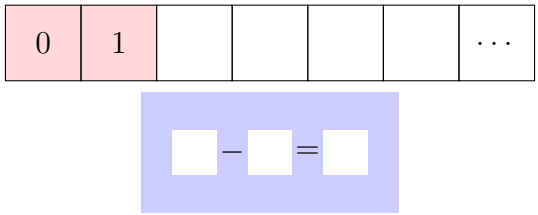
You can produce stripes and masks out of paper (or program a small animation) to try things out!

Q1: Can you find two different masks for the following sequence?



Q2: What happens to the Fibonacci sequence if you start with the values -1 and 1 instead? And what if you start with 0 and 0?

Q3: Which sequence do you get if you change the recursion of the Fibonacci sequence by turning the addition into a subtraction?



Q4: For periodic/arithmetic/geometric sequences, what is the mask to go backwards (i.e. to compute previous terms)?