

Unmasking Recurrence Sequences

There are many interesting sequences of numbers that can be described by some initial values and by a rule ('recurrence'). The **Fibonacci sequence**

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

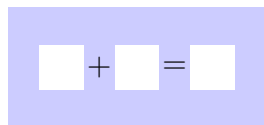
starts with the numbers 0 and 1. All other terms can be obtained with this simple rule:

Add the two previous terms.

Indeed, we have $0 + 1 = 1$, then $1 + 1 = 2$, then $1 + 2 = 3$, then $2 + 3 = 5$, and so on.

The sequence is a stripe of numbers, and the recurrence is a mask!

0	1	1	2	3	5	8	13	...
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You always see a true equality when the mask goes on the stripe!

0	1	1	2 + 3 = 5	8	13	...
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You can use the mask to compute the next term!

0	1	1	2 + 3 = □			
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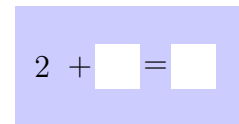
Periodic sequences are sequences that regularly repeat finitely many values. For example, consider the sequence with initial values 8,7, and 2, and with the rule "the value after three steps is the same":

8	7	2	8	7	2	8	7	...
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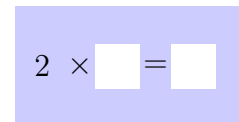
Arithmetic sequences require us to add one same number at each step. For example, consider the sequence with initial value 3 and with the rule "add 2":

3	5	7	9	11	13	15	...
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Geometric sequences require us to multiply by one same number at each step. For example, consider the sequence with initial value 3 and with the rule "multiply by 2":

3	6	12	24	48	96	192	...
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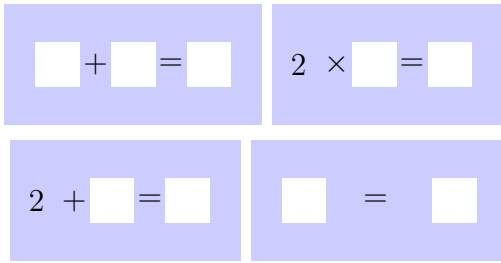
- You can produce stripes and masks out of paper (or program a small animation) to try things out...
- You have below four stripes and four masks. Can you associate them?

1	3	5	7	9	...
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1	2	1	2	1	...
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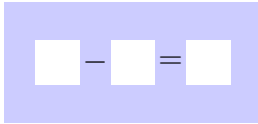
1	1	2	3	5	...
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1	2	4	8	16	...
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- Can you write down four more terms for the following sequence, using the given mask?

1	1					...
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- Can you figure out an appropriate mask for the following sequence?

2	4	16	256	...
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