

THE PYTEUK PUZZLE

MATHEMATICAL EXHIBIT BY ANTONELLA PERUCCA

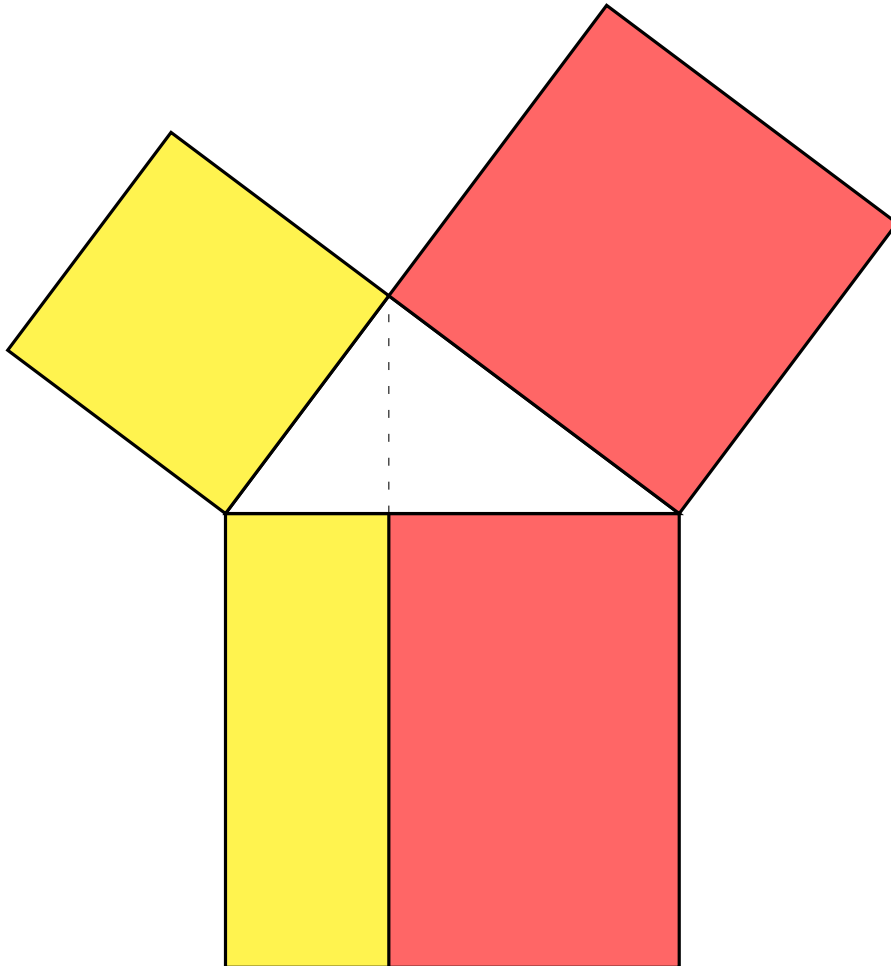


FIGURE 1. The puzzle shows more than Pythagoras' Theorem

There are several mathematical exhibits of various kind visualising Pythagoras' Theorem in a particular case. These exhibits do not show that the square on the hypotenuse, when cut by the altitude line, consists of two rectangles that have the size of the two squares on the cathetes. Both Pythagoras' Theorem and the stronger assertion can be visualized by the PytEuk puzzle for the right triangle with side-lengths 3,4, and 5 (the unit length can of course be arbitrarily chosen). Consider up to scaling the right triangle with side-lengths 15,20, and 25.

There are yellow pieces and red pieces. The yellow pieces form either a 15×15 square or a 9×25 rectangle. The red pieces form either a 20×20 square or a 16×25 rectangle. The 9×25 rectangle and the 16×25 rectangle taken together form a 25×25 square.

PYTEUK PUZZLE: THE 5-THEME

All pieces are rectangles such that one side has length 5.

The puzzle consists of 15 rectangular pieces (7 yellow pieces and 8 red pieces).

- The yellow pieces are: 3 pieces of size 9×5 , 2 pieces of size 6×5 , 2 pieces of size 3×5 .
- The red pieces are: 4 pieces of size 16×5 , 4 pieces of size 4×5 .

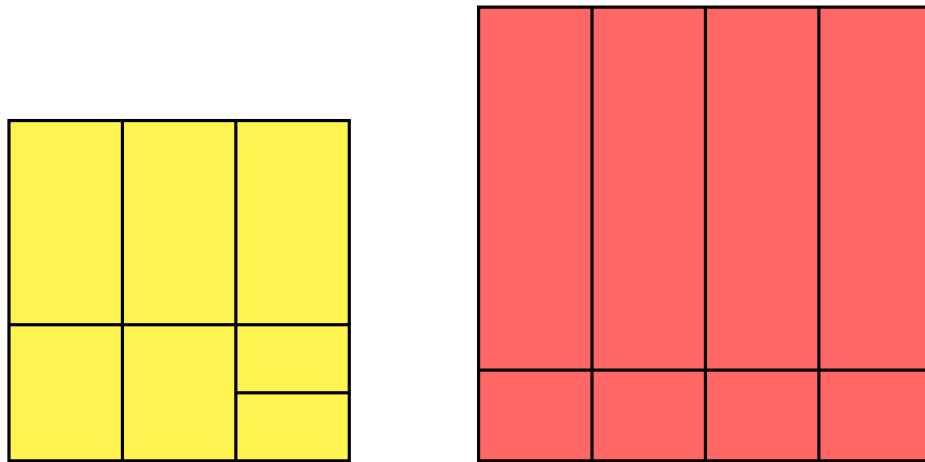


FIGURE 2. First possibility for the puzzle as two squares

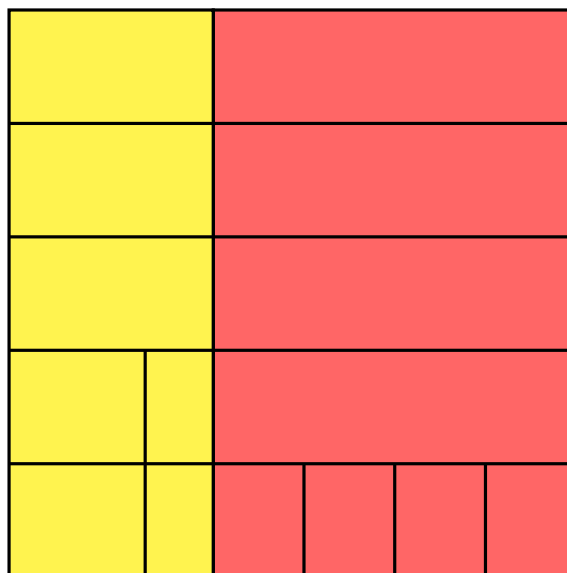


FIGURE 3. Second possibility for the puzzle as one square

PYTEUK PUZZLE: THE BIG-THEME

All pieces are rectangles which have been chosen to be large.

The puzzle consists of 9 rectangular pieces (4 yellow pieces and 5 red pieces).

- The yellow pieces are: 1 piece of size 9×15 , 1 pieces of size 6×10 , 2 pieces of size 3×5 .
- The red pieces are: 1 piece of size 16×20 , 4 pieces of size 10×5 .

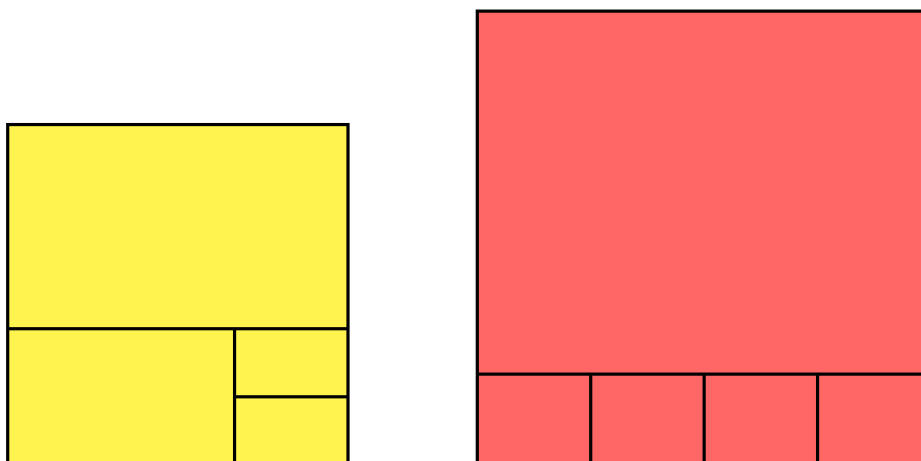


FIGURE 4. First possibility for the puzzle as two squares

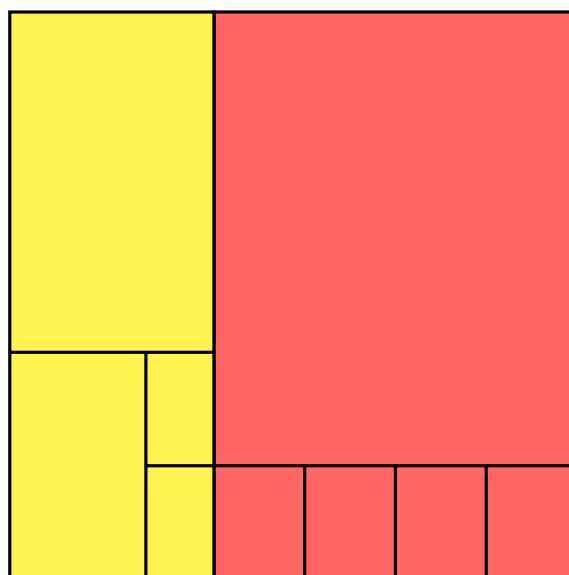


FIGURE 5. Second possibility for the puzzle as one square

PYTEUK PUZZLE: VARIANTS

We would restrict to **pieces which are rectangles and whose side-lengths are integers** (such pieces are very suitable to compare areas). Which variant is best depends on different criteria and it is also a matter of personal preference. For example one can try to *minimize the number of pieces*, possibly under the constraint that *all pieces must be squares*. Here is a possible variant:

The puzzle consists of 18 rectangular pieces (7 yellow pieces and 11 red pieces).

- The yellow pieces are: 3 pieces of size 9×5 , 2 pieces of size 6×5 , 2 pieces of size 3×5 .
- The red pieces are: 2 pieces of size 10×10 , 3 pieces of size 10×5 , 1 piece of size 5×5 , 5 pieces of size 1×5 .

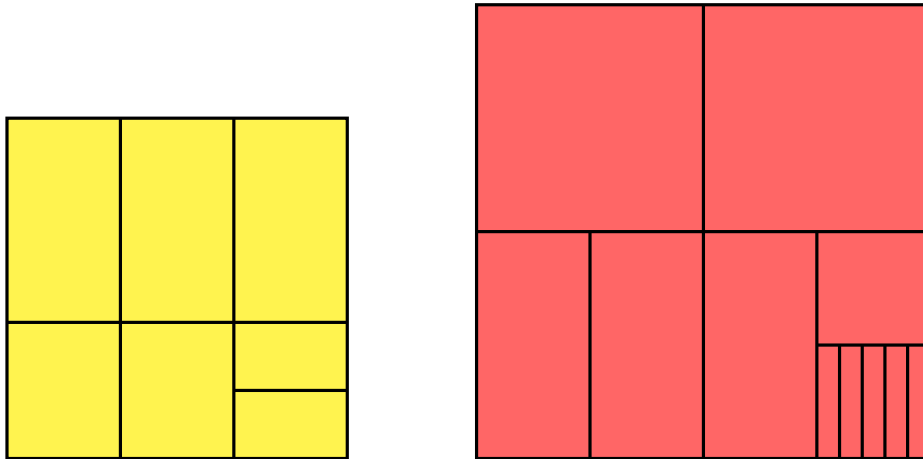


FIGURE 6. First possibility for the puzzle as two squares

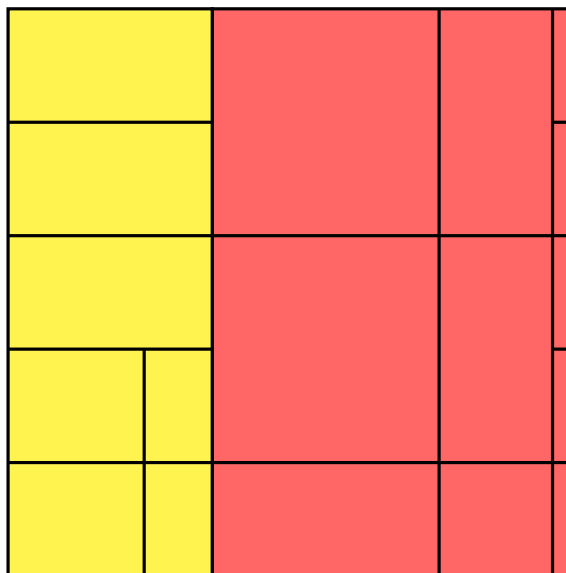


FIGURE 7. Second possibility for the puzzle as one square