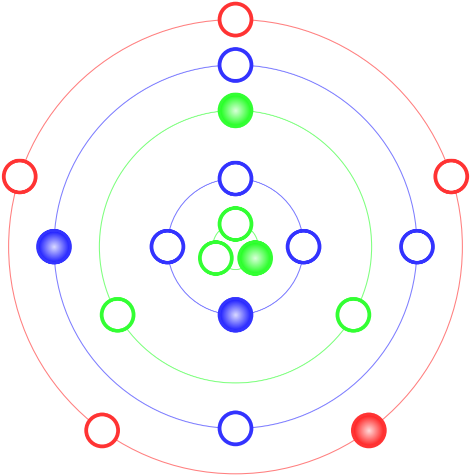
Chinese Remainder Clock



Hour: 3-remainder and 4-remainder

Number	0	1	2	3	4	5	6	7	8	9	10	11
3-remainder	0	1	2	0	1	2	0	1	2	0	1	2
4-remainder	0	1	2	3	0	1	2	3	0	1	2	3

A number from 0 to 11 is uniquely determined by its 3-remainder and 4-remainder.

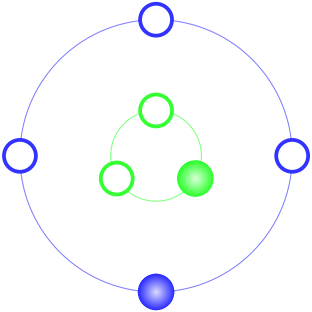
For example:

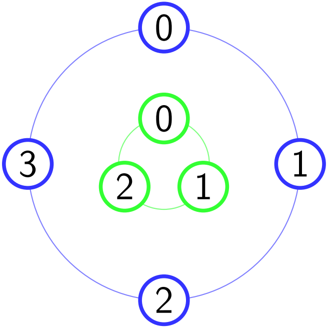
$$\text{3-remainder} = 2 \rightsquigarrow 2, 5, \mathbf{8}, 11$$

$$\text{4-remainder} = 0 \rightsquigarrow 0, 4, \mathbf{8}$$

Chinese Remainder Clock: Hour

Visualization of the 3-remainder and the 4-remainder:





Hour: one first algorithm

We are looking for a number X between 0 und 11.
Let R_3 and R_4 be the 3-remainder and the 4-remainder of X.

- X is one of the following numbers:

$$R_4 \qquad R_4 + 4 \qquad R_4 + 8$$

- Divide these numbers by 3 and choose the one whose 3-remainder equals R_3.

Hour: an alternative algorithm

We are looking for a number X between 0 und 11.
Let R_3 and R_4 be the 3-remainder and the 4-remainder of X.

- X is the 12-remainder of the number

$$4 \cdot R_3 + 9 \cdot R_4$$

- This number has the correct remainders:

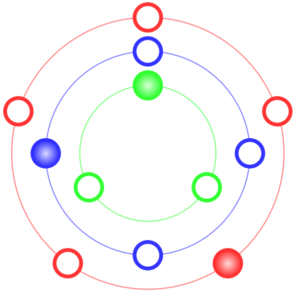
$$4 \cdot R_3 + 9 \cdot R_4 = 3 \cdot (R_3 + 3 \cdot R_4) + R_3$$

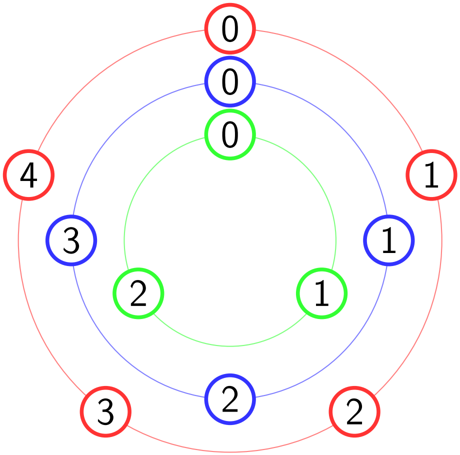
$$4 \cdot R_3 + 9 \cdot R_4 = 4 \cdot (R_3 + 2 \cdot R_4) + R_4$$

- A number and its 12-remainder have the same 3-remainder and the same 4-remainder.
- The 12-remainder is a number between 0 and 11.

Chinese Remainder Clock: Minute (Second)

Visualization of the 3-remainder, 4-remainder, and 5-remainder:





Minute (Second): one first algorithm

- **Even or odd:**
  A number and its 4-remainder have the same parity.
- **Last digit:**
  A number and its last digit have the same 5-remainder.

  If we know the 5-remainder of a number, and we know the parity of the number, then we know the last digit.
- **Minute (Second):**
  If we know the last digit, we have 6 possibilities for the minute. We take the number in this list that has the correct 3-remainder and 4-remainder.

Minute (Second): an alternative algorithm

We are looking for a number X between 0 und 59.
Let R_3, R_4 and R_5 be the 3-remainder, 4-remainder, and 5-remainder of X.

- X is the 60-remainder of the number

$$40 \cdot R_3 + 45 \cdot R_4 + 36 \cdot R_5$$

Indeed, we can write this number as follows:

$$3 \cdot (13 \cdot R_3 + 15 \cdot R_4 + 12 \cdot R_5) + R_3$$

$$4 \cdot (10 \cdot R_3 + 11 \cdot R_4 + 9 \cdot R_5) + R_4$$

$$5 \cdot (8 \cdot R_3 + 9 \cdot R_4 + 7 \cdot R_5) + R_5$$

Chinese Remainder Theorem

- Let $M_1, M_2, \ldots, M_n$ be pairwise coprime positive integers. Call P their product.
- If M is any number in the above list, choose freely one M-remainder, i.e. a number between 0 and $M-1$.
- Among any P consecutive integers, there is exactly one number that has the remainders that you have chosen.

We use this result as follows for the Chinese Remainder Clock:

Hour	$M_1 = 3,\ M_2 = 4$	$P = 12$
Minute (Second)	$M_1 = 3,\ M_2 = 4,\ M_3 = 5$	$P = 60$

Chinese Remainder Clock by Antonella Perucca, 2014.
Free for personal use (and use in class for teachers/lecturers).

Reference: *The Chinese Remainder Clock*, The College Mathematics Journal, 2017, Vol. 48, No. 2, pp. 82-89.